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The Australian Mathematical Society

Gazette

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- Mathematical articles of general interest, particularly historical and survey articles
- Reviews of books, particularly by Australian authors, or books of wide interest
- Classroom notes on presenting mathematics in an elegant way
- Items relevant to mathematics education
- Letters on relevant topical issues
- Information on conferences, particularly those held in Australasia and the region
- Information on recent major mathematical achievements
- Reports on the business and activities of the Society
- Staff changes and visitors in mathematics departments
- News of members of the Australian Mathematical Society

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Editorial

Welcome to the July issue of the *Gazette*.

In the Classroom Notes, Jerry Koliha and two of his third-year students, Sam Chow and Gus Schrader, describe how they approached a challenging problem in a special practice class in Linear Analysis. It is very encouraging to receive a contribution from undergraduate students, and we hope that both Sam and Gus and all their peers will send us many more articles in the future.

Also in this issue, Jon Borwein talks about exploratory experimentation in Maths Matters, a topic close to his heart. We also continue our new series Mathematical Minds with an interview with Rob McIntosh from Telstra's Chief Technology Office. And Nalini Joshi, Hyam Rubenstein and Philip Broadbridge all ponder the issues of funding for Australian mathematics.

Congratulations to Ivan Guo for winning the latest Puzzle Corner book voucher. And on behalf of all our readers, congratulations to Dr Norman Do for completing his PhD while contributing the regular Puzzle Corner.

Happy reading from the *Gazette* team.

Vacancy: Editors for the ***Gazette***

The present Editors of the *Gazette*, Birgit Loch and Rachel Thomas, are, sadly, stepping down from their position on 31 December 2009. So the Society is looking for new Editors for the *Gazette*. An overlap in the position of a few months, from about October 2009, is envisaged, to enable a smooth transition to the new editors.

Anyone interested in the position of Editor is invited to send (via e-mail) a brief resumé and covering letter to both the President and the Secretary, at President@austms.org.au and Secretary@austms.org.au.

The current Production Editor, Eileen Dallwitz, is expected to continue, so there is no need for the incoming Editors to know any TEX although such knowledge would still be an advantage. There will be some financial assistance provided towards teaching relief and/or AustMS conference registration and expenses.

For further information about what the position entails, please contact the present Editors at gazette@austms.org.au .

Elizabeth J Billington
Hon. Sec., AustMS

President's column

Nalini Joshi*

I have spent the last two months at the Isaac Newton Institute for Mathematical Sciences at Cambridge University in the UK, in mathematical heaven. International centres like the Newton Institute operate six-month-long[1] research programs to bring together groups of people with similar interests, who arrive and depart at different times, whose interactions and questions constantly inject new ideas into on-going discussions. Out of such mergers grow beautiful ideas and, sometimes, breakthroughs.

For mathematics to grow, conversations about ideas and new techniques are essential. Individual conversations happen all the time, whether in your departmental corridors and tearooms, in seminars, workshops and conferences, in conversation with invited visitors, or in sabbaticals with your hosts and members of the hosting group. Shorter conversations may spark new, and sometimes very deep, ideas. Longer conversations, carried out at irregular intervals over email, Skype, and through visits, develop layers of ideas that sometimes germinate only years later. Sustained, extended conversations with people who are attracted to the same open questions as you, and whom you meet serendipitously at an extended program, lead to an unlocking of the mind.

Unfortunately, Australia has no funding mechanism that can support six-month-long research programs on different themes in the mathematical sciences. We cannot bring together a critical mass of leading mathematical scientists from Australia and overseas to interact over extended periods. We cannot support junior mathematicians for extended periods to meet and interact with leading scientists in their area. We do not have a mechanism that encourages and supports high-quality research programs that range over all areas of mathematics.

The Australian Research Council[2] recognises the necessity of some collaborative mechanisms for research. Certainly, attendance at conferences is supported. Support for individual collaborative visitors is now available for those working on specific projects. However, all of the ARC's funding is focused on individual projects and programs that involve one or at most two themes. In the last two decades, the ARC has funded several Special Research Centres and Centres of Excellence. Whilst the aims were lofty, in practice the Centres were required to focus on specific areas of research. This meant that one could not mount a program in say algebraic Lie theory in one semester and change to a program in random matrix theory in the next.

Another constraint is the way in which accountability became synonymous with commercial application. The ARC Centres of Excellence Funding Rules for 2005

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[1]They also have shorter programs in the summer months.
[2]See guidelines at http://www.arc.gov.au.

state (on page 6)[3]: 'ARC Centres of Excellence are likely to make discoveries that have the potential for development to the point of commercial application'. Successive six-month programs in the mathematical sciences are unlikely to yield direct commercial applications. Even a program on *L-functions and arithmetic*[4] may have no short-term commercialisation gains, although the application of number theory to secure communications technology is vast.

A different argument needs to be put to the ARC to show the benefit of a national institute of research in mathematical sciences. Its activities will support research in a large cross-section of areas across Australia, rather than focus support on a group of individuals. It is, in fact, an efficient way of distributing research funding for the nation's needs. It will also provide research support for junior researchers who are currently not being as well supported as our senior researchers.

The Steering Committee has been conversing on this topic. In our second meeting this year by phone, Leanne Harvey and Andrew Calder from the ARC joined us, primarily to talk about the bibliometric processes underlying the ERA. However, we also talked briefly about the necessity for a national centre in the mathematical sciences. Leanne Harvey made a very good point, that higher education research funding is done on a dual funding process: the ARC deals with project based funding, while the block funding mechanism is done through the Department of Innovation, Industry, Science and Research. So our needs fall between two stools.

There is another sobering thought here, which is that we need the support and cooperation of other sciences in Australia to achieve our aims. In England, the Higher Education Funding Council has recently selected the University of Birmingham to host the national higher education program for science, technology, engineering and mathematics (STEM) to the tune of 20M pounds. In the UK, it seems disciplines work together. High-profile scientists in Australia know that the deterioration of mathematical sciences hurts their students and want to fix this problem. One idea that has been put forward is that we form a new group together with such scientists to have a high-level dialogue on the issues of concern for all of us. My hope is that, with the agreement across the sciences, we can present our case to government for support of the kind we need.

Nalini Joshi holds a PhD and MA from Princeton University in Applied Mathematics and a BSc (Hons) from the University of Sydney. In 2002, she returned to the University of Sydney to take up the Chair of Applied Mathematics and became the first female mathematician to hold a Chair there. In 2008, she was elected a Fellow of the Australian Academy of Science. She is currently the Head of the School of Mathematics and Statistics. Her research focuses on longstanding problems concerning the asymptotic and analytic structure of solutions to non-linear inte- grable equations.

[3]On the same page, it is stated: 'Centres must foster amongst their staff an awareness of sound innovation and commercialisation practice, and will encourage entrepreneurial activity in appropriate circumstances.'

[4]This was the name of a program at the Newton Institute in 1993 at which Wiles revealed his proof of Fermat's last theorem.

Letter to the editors

Efficient, national, computerised assessement for the Tertiary Education Quality and Standards Agency: Well, for the maths and stats assessment anyway

In this year's Budget, a Tertiary Education Quality and Standards Agency was proposed[1].

There are internet-delivered computer-aided assessment methods of testing students (and staff if you like). I'm tolerably knowledgeable about those in mathematics. There is an open-source system (the computer-aided assessment package STACK[2], underpinned by the virtual-learning environment Moodle[3]) which would be particularly appropriate for the TEQS-Agency.

This isn't a deliberate plug for the made-in-WA component of it, namely Moodle. The system really does work. It is about to be in use (or possibly has just started to be used) in use at the British Open University.

Basically, the TEQS-Agency could run STACK/Moodle on several servers and be delivered in each of the capital cities, probably in rooms in the maths buildings of a major university in each state capital. (The multiple servers may be necessary to cope with large loads, which could occur if a class of 500–1000, say, had coinciding due dates and times.)

STACK/Moodle would deliver out questions for the students to do, mark them and give instant feedback. The universities could choose their own question banks appropriate to their units (but tell the TEQS-Agency) and have these either for student practice or even as part of the assessment of the course. The students would probably do these questions on home computers after getting used to the system at a computer-lab class on campus. The TEQS-Agency could run invigilated quizzes based on the system in the computer labs of the universities to obtain more reliable data on the skill levels of the students. (It is all very well the students doing the work at home, but there is at least a theoretical possibility of them having a friend do the questions for them.) The TEQS-Agency would have the data on the year of study of the students, their major and their university, and would be able to compare their achievements on the same questions across the different universities.

STACK can be trialled via guest access at http://stack.bham.ac.uk/. Documents on its use at the Open University include

[1] http://www.deewr.gov.au/Ministers/Gillard/Media/Releases/Pages/Article_090512_182729.aspx
[2] http://www.stack.bham.ac.uk/
[3] http://moodle.org/

www.open.ac.uk/cetl-workspace/cetlcontent/documents/49e85cfa5d4ea.pdf and www.caaconference.co.uk/pastConferences/2008/proceedings/Butcher_P_final_formatted_n1.pdf.

It should be emphasised that I see systems like this as useful for the students' education too. Indeed a system I liked (but which contains an over-priced commercial component) has as part of it a free open-source component called ALICE (assisted learning in a computer environment). The instant feedback and multiple attempts it allows work well to help students learn. At large universities there are lots of options other than STACK/Moodle and here in Perth, at UWA and at Curtin, we are using other computer-aided assessment systems. Thus I have no actual experience with using STACK with classes. I know STACK's author. However, STACK/Moodle works (and is free and open-source).

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Classroom notes

A measure blowup

Sam Chow* **, Gus Schrader*** *** and J.J. Koliha*** ****

Last year Sam Chow and Gus Schrader were third-year students at the University of Melbourne enrolled in Linear Analysis, a subject involving — among other topics — Lebesgue measure and integration. This was based on parts of the recently published book *Metrics, Norms and Integrals* by the lecturer in the subject, Associate Professor Jerry Koliha. In addition to three lectures a week, the subject had a weekly practice class. The lecturer introduced an extra weekly practice class with a voluntary attendance, but most students chose to attend, showing a keen interest in the subject. The problems tackled in the extra practice class ranged from routine to very challenging. The present article arose in this environment, as Sam and Gus independently attacked one of the challenging problems.

Working in the Euclidean space $\mathbb{R}^d$ we can characterise Lebesgue measure very simply by relying on the Euclidean volume of the so-called *d-cells.* A d-cell C in $\mathbb{R}^d$ is the Cartesian product

$$C = I_1 \times \cdots \times I_d$$

of bounded nondegenerate one-dimensional intervals I_k; its Euclidean d-volume is the product of the lengths of the I_k. *Borel sets* in $\mathbb{R}^d$ are the members of the least family $\mathcal{B}^d$ of subsets of $\mathbb{R}^d$ containing the empty set, $\emptyset$, and all d-cells which are closed under countable unions and complements in $\mathbb{R}^d$. Since every open set in $\mathbb{R}^d$ can be expressed as the countable union of suitable d-cells, the open sets are Borel, as are all sets obtained from them by countable unions and intersections, and complements. The Lebesgue measure $m = m_d$ is a countably additive set function $m\colon \mathcal{B}^d \to [0,\infty]$ satisfying $m(\emptyset) = 0$ and coinciding with the Euclidean d-volume on d-cells; such a function is unique and monotonic with respect to the set inclusion. We do not need to know how to calculate the Lebesgue measure of a Borel set to get quite far on this characterisation alone.

The faces of d-cells are Borel sets and their d-dimensional Lebesgue measure is zero; that much can be obtained by embedding them in suitable d-cells and using the monotonicity of the Lebesgue measure. In other words, the boundaries of d-cells

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have Lebesgue measure zero. It is tempting to think that open sets in $\mathbb{R}^d$ (countable unions of d-cells) also have boundaries of Lebesgue measure zero. There do not seem to be obvious open set candidates to counter this.

However, Sam came up with this ingenious example.

Example 1. Let S be the 'fat Cantor set' obtained by removing the open middle quarter from the interval $[0,1]$, and then proceeding to remove the open subinterval of length $(\frac{1}{2})^{2n}$ from the middle of each of 2^{n-1} remaining intervals for $n = 2, 3, \ldots$. Then $G_1 = [0,1] \setminus S$ is open in $\mathbb{R}$, and the Lebesgue measure of G is $m(G_1) = \sum_{n=1}^{\infty} 2^{n-1}(\frac{1}{2})^{2n} = \frac{1}{2}$. Then $m(S) = \frac{1}{2}$, while S is the boundary of G_1. The set $G = G_1 \times (0,1) \times \cdots \times (0,1)$ (with $d-1$ factors $(0,1)$) is open in $\mathbb{R}^d$ with the boundary $\partial G = S \times (0,1) \times \cdots \times (0,1)$, where $m(G) = \frac{1}{2} = m(\partial(G))$.

Gus found an even more dramatic result.

Theorem 1. (i) *Given $\varepsilon > 0$ there exists an open set $G \subset \mathbb{R}^d$ with $m(G) \leq \varepsilon$ and $m(\partial G) = \infty$.*

(ii) *Given $\varepsilon > 0$ and $0 < M < \infty$, there exists a bounded open set $G \subset \mathbb{R}^d$ with $m(G) \leq \varepsilon$ and $m(\partial G) \geq M$.*

Proof. Let $\varepsilon > 0$, $0 < M \leq \infty$, and let F be a closed subset of $\mathbb{R}^d$ with Lebesgue measure $m(F) \geq M + \varepsilon$; this is always possible ($\infty + \varepsilon = \infty$ by convention). Let A be the set of all points in F with rational coordinates; this set is countable and dense in F, that is, $\overline{A} = F$. We can order the points of A in a sequence, $A = \{q_n \colon n = 1, 2, \ldots\}$. Let B_n be an open d-cell in $\mathbb{R}^d$ containing q_n whose Euclidean volume is $(\frac{1}{2})^n\varepsilon$ (give an explicit construction of B_n — if you are so inclined). Then $G = \bigcup_{n=1}^{\infty} B_n$ is an open set whose Lebesgue measure satisfies

$$m(G) \leq \sum_{n=1}^{\infty} m(B_n) = \sum_{n=1}^{\infty} \left(\tfrac{1}{2}\right)^n \varepsilon = \varepsilon.$$

Let $\overline{G}$ be the closure of G. Then $\overline{G} = G \cup \partial G$ is a disjoint union, and

$$m(\overline{G}) = m(G) + m(\partial G).$$

From $F = \overline{A} \subset \overline{G}$ it follows that $m(\overline{G}) \geq m(F) \geq M + \varepsilon$. Assertion (i) follows when we choose $M = \infty$: then $m(\overline{G}) = \infty = m(\partial G)$. To prove (ii) with finite M and bounded G, we choose F in the foregoing argument bounded and satisfying $m(F) \geq M + \varepsilon$. Then G, $\overline{G}$ and ∂G are also bounded and therefore of finite Lebesgue measure, so that $m(\partial G) = m(\overline{G}) - m(G) \geq (M + \varepsilon) - \varepsilon = M$.

Note: When requiring G in part (i) of the preceding theorem to be bounded we can no longer achieve $m(\partial G) = \infty$.

For more detail on the 'fat Cantor set' see Wikipedia
http://en.wikipedia.org/wiki/Smith-Volterra-Cantor_set.

Both Sam and Gus feel that although the Linear Analysis course was one of the most challenging they had taken, the enthusiasm and encouragement of the lecturer, and the opportunity to tackle challenging problems in the practice classes, made what would be an initimidating subject more accessible. Their experience of independent research in these practical classes led them to a greater appreciation of the subtleties of a subject as rich as analysis, and improved their understanding of mathematics dramatically.

Sam is a pure maths student at the University of Melbourne, hoping to study honours next year. He has always loved mathematical problem solving, and represented Australia in the 2005 IMO. Chess is his main hobby, often travelling overseas to play in tournaments. Other hobbies include soccer, squash, badminton and poker.

Gus Shrader is originally from Adelaide. At the time this note was written he was a third-year maths major at the University of Melbourne. He is now an Honours student in pure maths, and his Honours project is on algebraically completely integrable systems, in particular the so-called Hitchin systems.

Jerry Koliha is Associate Professor in the Mathematics and Statistics Department of the University of Melbourne. He specialises in Functional Analysis. He says that every year he learns from his students as much as they learn from him. At the end of this year, Jerry will be retiring from the University of Melbourne after 40 uninterrupted years. He can think of no better finish to his career than knowing he has inspired his students to appreciate mathematical analysis so much.

Exploratory experimentation: digitally-assisted discovery and proof

Jonathan M. Borwein*

Exploratory experimentation

Our community (appropriately defined) is facing a great challenge to re-evaluate the role of proof in light of the growing power of current computer systems, of modern mathematical computing packages and of the growing capacity to data-mine on the internet. Add to that the enormous complexity of many modern mathematical results such as the Poincaré conjecture, Fermat's last theorem, and the classification of finite simple groups. As the need and prospects for inductive mathematics blossom, the need to ensure that the role of proof is properly founded remains undiminished. I share with Pólya the view that

> [I]ntuition comes to us much earlier and with much less outside influence than formal arguments ... Therefore, I think that in teaching high school age youngsters we should emphasize intuitive insight more than, and long before, deductive reasoning.
>
> George Pólya (1887–1985) [15, p. 128]

He goes on to reaffirm, nonetheless, that proof should certainly be taught in school. I continue with some observations, many of which have been fleshed out in my recent books *The Computer as Crucible* [7], *Mathematics by Experiment* [6] and *Experimental Mathematics in Action* [3]. My musings focus on the changing nature of mathematical knowledge and in consequence ask questions such as 'How do we come to believe and trust pieces of mathematics?', 'Why do we wish to prove things?' and 'How do we teach what and why to students?'. I am persuaded by various notions of embodied cognition. Smail [16, p. 113] writes: '[T]he large human brain evolved over the past 1.7 million years to allow individuals to negotiate the growing complexities posed by human social living.' In consequence we find various modes of argument more palatable than others, and are more prone to make certain kinds of errors than others. Likewise, Steve Pinker's observation about language [14, p. 83] as founded on '... the ethereal notions of space, time, causation, possession, and goals that appear to make up a language of thought' remains equally potent within mathematics. The computer offers to provide scaffolding both to enhance mathematical reasoning and to restrain mathematical error.

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This material is largely excerpted from a longer paper written for ICMI Study 19 'On Proof and Proving in Mathematics Education', and a plenary talk given in May 2009 at the corresponding Workshop at National Taiwan Normal University.

To begin with, let me briefly reprise what I mean by discovery, and by proof. Marcus Giaquinto [10, p. 50] writes: 'discovering a truth is coming to believe it in an independent, reliable, and rational way'. This attractive notion of *discovery* has the satisfactory consequence that a student can certainly discover results whether known to the teacher or not. Nor is it necessary to demand that each dissertation be original (only independently discovered). Next I define:

> **Proof**[1], n. a sequence of statements, each of which is either validly derived from those preceding it or is an axiom or assumption, and the final member of which, the conclusion, is the statement of which the truth is thereby established.

As for mathematics itself, I offer the following in which the term *proof* does not enter. Nor should it; mathematics is much more than proof alone:

> **Mathematics**, n. a group of subjects, including algebra, geometry, trigonometry and calculus, concerned with number, quantity, shape, and space, and their inter-relationships, applications, generalizations and abstractions.
>
> **Deduction**, n. the process of reasoning typical of mathematics and logic, in which a conclusion follows necessarily from given premises so that it cannot be false when the premises are true.
>
> **Induction**, n. (Logic) a process of reasoning in which a general conclusion is drawn from a set of particular premises, often drawn from experience or from experimental evidence. The conclusion goes beyond the information contained in the premises and does not follow necessarily from them. ... for example, large numbers of sightings at widely varying times and places provide very strong grounds for the falsehood that all swans are white.

It awaited the discovery of Australia to confound the seemingly compelling inductive conclusion that all swans are white. I observe that we typically take for granted the distinction between *induction* and *deduction* and rarely discuss their roles with either our colleagues or our students.

Despite the conventional identification of mathematics with deductive reasoning, in his 1951 Gibbs Lecture Kurt Gödel (1906–1978) said: 'If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics.' He held this view until the end of his life despite the epochal deductive achievement of his incompleteness results. Moreover, one discovers a substantial number of great mathematicians, from Archimedes and Galileo — who apparently said 'All truths are easy to understand once they are discovered; the point is to discover them' — to Poincaré and Carleson, who have emphasised how much it helps to 'know' the answer. Over two millennia ago Archimedes wrote to Eratosthenes in the introduction to his long-lost and recently re-constituted *Method of Mechanical Theorems* [12]:

[1] All definitions are taken from the *Collin's Dictionary of Mathematics* which I co-authored. It is freely available within the MAA's digital mathematics library and commercially with Maple inside it at http://www.mathresources.com/products/mathresource/index.html

> For some things, which first became clear to me by the mechanical method, were afterwards proved geometrically, because their investigation by the said method does not furnish an actual demonstration. For it is easier to supply the proof when we have previously acquired, by the method, some knowledge of the questions than it is to find it without any previous knowledge.

Think of the *Method* as an ur-precursor to today's interactive geometry software — with the caveat that, for example, the interactive geometry software package *Cinderella* actually does provide certificates for much Euclidean geometry.

As 2006 Abel Prize winner Leonard Carleson described in his 1966 ICM speech on his positive resolution of Luzin's 1913 conjecture, after many years of seeking a counter-example he decided none could exist. The importance of this confidence is expressed as follows:

> The most important aspect in solving a mathematical problem is the conviction of what is the true result. Then it took 2 or 3 years using the techniques that had been developed during the past 20 years or so.

I will now assume that all proofs discussed are 'non-trivial' in some fashion — since the issue of using inductive methods is really only of interest with this caveat. Armed with these terms, it remains to say that by *digital assistance* I intend the use of such *artefacts* as:

(1) *Modern mathematical computer packages* — be they symbolic, numeric, geometric, or graphical. I would classify all of these as 'modern hybrid workspaces'. One might also envisage much more use of stereo visualisation, haptic, and auditory devices.
(2) *More specialist packages* or *general purpose languages* such as Fortran, C++, CPLEX, GAP, PARI, SnapPea, Graffiti, and MAGMA.
(3) *Web applications* such as: Sloane's Online Encyclopedia of Integer Sequences, the Inverse Symbolic Calculator, Fractal Explorer, Jeff Weeks' Topological Games, or Euclid in Java[2].
(4) *Web databases* including Google, MathSciNet, ArXiv, Wikipedia, JSTOR, MathWorld, Planet Math, Digital Library of Mathematical Functions or (DLMF), MacTutor, Amazon, and many more sources that are not always viewed as part of the palette.

All entail *data-mining* in various forms. Franklin [9] argues that what Steinle calls '*exploratory experimentation*' facilitated by '*widening technology*' as in pharmacology, astrophysics, biotechnology is leading to a reassessment of what is viewed as a legitimate experiment; in that a '*local model*' is not a prerequisite for a legitimate experiment. Hendrik Sørenson [17] cogently makes the case that *experimental mathematics* — as 'defined' below — is following similar tracks.

> These aspects of exploratory experimentation and wide instrumentation originate from the philosophy of (natural) science and have not been much developed in the context of experimental mathematics. However, I claim that

[2]A cross-section of such resources is available through http://ddrive.cs.dal.ca/~isc/portal.

> e.g. the importance of wide instrumentation for an exploratory approach to experiments that includes concept formation also pertain to mathematics.

Danny Hillis is quoted as saying recently that: 'Knowing things is very 20th century. You just need to be able to find things.' on how *Google* has already changed how we think[3]. This is clearly not yet true and will never be, yet it catches something of the changing nature of cognitive style in the 21st century. In consequence, the boundaries between mathematics and the natural sciences and between inductive and deductive reasoning are blurred and getting blurrier. This is discussed at some length by Jeremy Avigad [1].

Experimental mathodology

We started [7] with Justice Potter Stewart's famous 1964 comment on pornography 'I know it when I see it'. I now reprise from [6] what we mean a bit less informally by *experimental mathematics*. Gaining insight and *intuition*; *Discovering* new relationships; *Visualising* math principles; *Testing* and especially *falsifying* conjectures; *Exploring* a possible result to see if it *merits* formal proof; *Suggesting* approaches for formal proof; *Computing* replacing lengthy hand derivations; *Confirming* analytically derived results. Of these the first five play a central role and the sixth plays a significant one but refers to computer-assisted or computer-directed proof and is quite far from *Formal Proof* as the topic of a special issue of the *Notices of the AMS* in December 2008.

Mathematical examples

I continue with various explicit examples. I leave it to the reader to decide how frequently he or she wishes to exploit the processes I advertise.

Example I: what did the computer do?

In my own work computer experimentation and digitally-mediated research now invariably play a crucial part. (Even in seemingly non-computational areas of functional analysis and the like, there is frequently a computable consequence whose verification provides confidence in the result under development.) In a recent study of expectation or 'box integrals' [4] we were able to evaluate a quantity, which had defeated us for years, namely

$$\mathcal{K}_1 := \int_3^4 \frac{\operatorname{arcsec}(x)}{\sqrt{x^2 - 4\,x + 3}}\,\mathrm{d}x,$$

in *closed-form* as

$$\mathcal{K}_1 = \mathrm{Cl}_2(\theta) - \mathrm{Cl}_2\left(\theta + \frac{\pi}{3}\right) - \mathrm{Cl}_2\left(\theta - \frac{\pi}{2}\right) + \mathrm{Cl}_2\left(\theta - \frac{\pi}{6}\right) - \mathrm{Cl}_2\left(3\,\theta + \frac{\pi}{3}\right)$$

[3] In Achenblog http://blog.washingtonpost.com/achenblog/ of 1 July 2008.

$$+\,\mathrm{Cl}_2\left(3\,\theta+\frac{2\pi}{3}\right)-\mathrm{Cl}_2\left(3\,\theta-\frac{5\pi}{6}\right)+\mathrm{Cl}_2\left(3\,\theta+\frac{5\pi}{6}\right)$$
$$+\left(6\,\theta-\frac{5\,\pi}{2}\right)\log\left(2-\sqrt{3}\right), \tag{1}$$

where $\mathrm{Cl}_2(\theta) := \sum_{n=1}^{\infty} \sin(n\theta)/n^2$ is the *Clausen function*, and

$$3\,\theta := \arctan\left(\frac{16-3\sqrt{15}}{11}\right)+\pi.$$

Along the way to the evaluation above, there were several stages of symbolic computation, including an expression for $\mathcal{K}_1$ with over 28 000 characters (perhaps 25 standard novel pages). It may well be that the closed form in (1) can be further simplified. In any event, the very satisfying process of distilling the computer's 28 000 character discovery, required a mixture of art and technology and I would be hard pressed to assert categorically whether it constituted a conventional proof. Nonetheless, it is correct and has been checked numerically to over a thousand-digit decimal precision.

I turn to an example I hope will reinforce my assertion that there is already an enormous amount to be mined on the internet. And this is before any mathematical character recognition tools have been made generally available and when it is still very hard to search mathematics on the web.

Example II: what is that number?

In 1995 or so Andrew Granville emailed me the number

$$\alpha := 1.4331274267223\ldots \tag{2}$$

and challenged me to identify it; I think this was a test I could have failed. I asked *Maple* for its continued fraction. In the conventional concise notation I was rewarded with

$$\alpha = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots]. \tag{3}$$

Even if you are unfamiliar with continued fractions, you will agree that the changed representation in (3) has exposed structure not apparent from (2)! I reached for a good book on continued fractions and found the answer $\alpha = I_1(2)/I_0(2)$ where I_0 and I_1 are *Bessel functions* of the first kind. Actually I remembered that all arithmetic continued fractions arise in such fashion, but as we shall see one now does not need to. In 2009 there are at least three 'zero-knowledge' strategies: 1. Given (3), type 'arithmetic progression', 'continued fraction' into *Google*; 2. Type '1,4,3,3,1,2,7,4,2' into *Sloane's Encyclopaedia of Integer Sequences*[4]; 3. Type the decimal digits of α into the *Inverse Symbolic Calculator*[5]. I illustrate the results of each strategy.

[4] See http://www.research.att.com/~njas/sequences/.
[5] The online *Inverse Symbolic Calculator* http://ddrive.cs.dal.ca/~isc was newly web-accessible in 1995.

1. The first three hits on typing ‘arithmetic progression’, ‘continued fraction’ into *Google* on 15 October 2008, are shown in Figure 1. Moreover, the MathWorld entry tells us that any arithmetic continued fraction is of a ratio of Bessel functions, as shown in the inset to Figure 1 which points to the second hit in Figure 1. The reader may wish to find which other search terms uncover the answer — perhaps in the newly unveiled *Wolfram Alpha*.

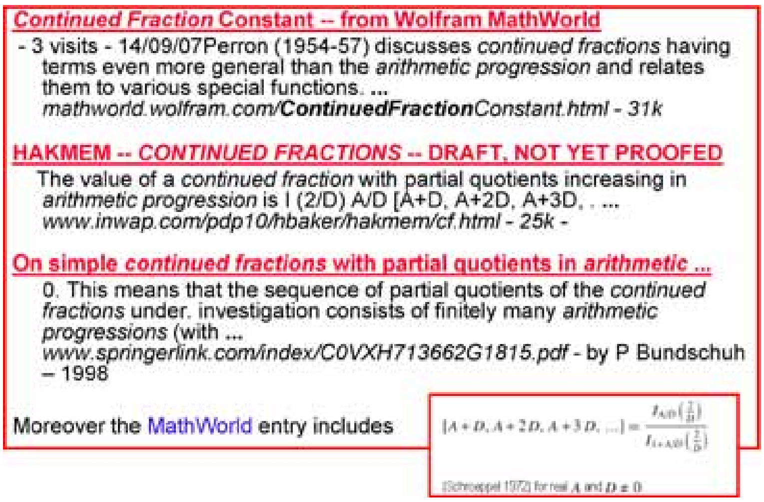

Figure 1. What Google and MathWorld offer.

2. Typing the first few digits into Australian expat Sloane’s interface yields Figure 2. In this case we are even told what the series representations of the requisite Bessel functions are, we are given sample code (in this case in *Mathematica*), and we are lead to many links and references. Moreover, the site is carefully moderated and continues to grow. This strategy became viable after 14 May 2008 when the sequence was added to the database — now with over 158 000 entries.

3. If one types the decimal for α into the Inverse Symbolic Calculator (ISC) it returns `Best guess:  BesI(0,2)/BesI(1,2)`.

Most of the functionality of the ISC is built into the ‘identify’ function in *Maple* starting with version 9.5. For example, `identify(4.45033263602792)` returns $\sqrt{3}+e$. As always, the experienced will extract more than the novice.

Example III: from discovery to proof

The following was popularised in *Eureka*[6] in 1971.

$$0 < \int_0^1 \frac{(1-x)^4 x^4}{1+x^2}\,\mathrm{d}x = \frac{22}{7} - \pi \qquad (4)$$

[6]Eureka was an undergraduate Cambridge University journal.

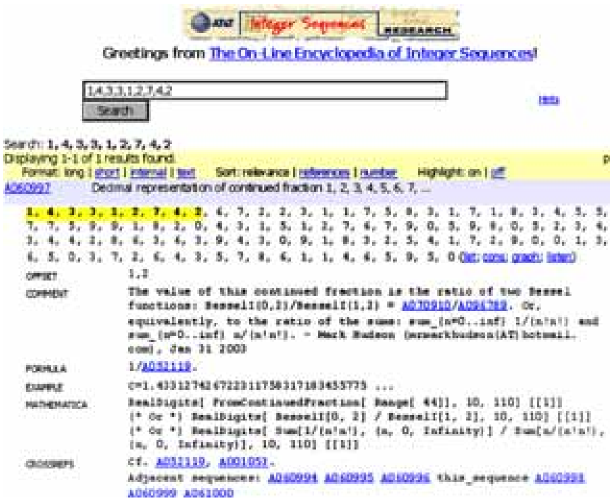

Figure 2. What *Sloane's Encyclopaedia* offers.

as described in [6]. The integrand is positive on $(0,1)$ so the integral yields an area and $\pi < 22/7$. Set on a 1960 Sydney honours maths final exam[7] (4) perhaps originated in 1941 with the author of the 1971 article — Dalzeil who chose not reference his earlier self! Why should we trust this discovery? Well *Maple* and *Mathematica* both 'do it'. But this is *proof by appeal to authority* [11] and a better answer is to ask *Maple* for the indefinite integral

$$\int_0^{\mathbf{t}} \frac{(1-x)^4 x^4}{1+x^2}\, \mathrm{d}x = ?$$

The computer algebra system (CAS) will return

$$\int_0^t \frac{x^4(1-x)^4}{1+x^2}\, \mathrm{d}x = \frac{1}{7}\, t^7 - \frac{2}{3}\, t^6 + t^5 - \frac{4}{3}\, t^3 + 4\, t - 4\, \arctan(t). \tag{5}$$

Finish by differentiating and using the Fundamental theorem of calculus.

This is probably not the proof one would find by hand, but it is rigorous, and represents an 'instrumental use' of the computer. That a CAS will often be able to evaluate an indefinite integral or a finite sum whenever it can evaluate the corresponding definite integral or infinite sum frequently allows one to provide a certificate for such a discovery. In the case of a sum the certificate often takes the form of a mathematical induction. Another interesting feature of Example III is that it appears to be irrelevant that 22/7 is the most famous continued-fraction approximation to π, as described by an expat Australian [13]. Not every discovery is part of a hoped-for pattern.

[7] Alf van der Poorten recalls being shown this by Kurt Mahler in the mid-sixties.

Example IV: from concrete to abstract

While studying *multiple zeta values* [6] we needed to show $M := A + B - C$ invertible, where the $n \times n$ matrices A, B, C respectively had entries

$$(-1)^{k+1}\binom{2n-j}{2n-k}, \qquad (-1)^{k+1}\binom{2n-j}{k-1}, \qquad (-1)^{k+1}\binom{j-1}{k-1}. \tag{6}$$

So A and C are triangular while B is full. In six dimensions M is:

$$\begin{bmatrix} 1 & -22 & 110 & -330 & 660 & -924 \\ 0 & -10 & 55 & -165 & 330 & -462 \\ 0 & -7 & 36 & -93 & 162 & -210 \\ 0 & -5 & 25 & -56 & 78 & -84 \\ 0 & -3 & 15 & -31 & 35 & -28 \\ 0 & -1 & 5 & -10 & 10 & -6 \end{bmatrix}.$$

After futilely peering at many cases I thought to ask *Maple* for the *minimal polynomial* of M:

```
> linalg[minpoly](M(12),t);
```

returns $-2 + t + t^2$. Emboldened I tried

```
> linalg[minpoly](B(20),t); linalg[minpoly](A(20),t);
  linalg[minpoly](C(20),t);
```

and was rewarded with $-1 + t^3$, $-1 + t^2$, $-1 + t^2$. A typical matrix has a full degree minimal polynomial, so we are assured that A, B, C really are roots of unity. Armed with this we are lead to try to prove

$$A^2 = I, \qquad BC = A, \qquad C^2 = I, \qquad CA = B^2, \tag{7}$$

which is a nice combinatorial exercise (by hand or computer). Clearly then we obtain also

$$B^3 = B \cdot B^2 = B(CA) = (BC)A = A^2 = I \tag{8}$$

and $M^{-1} = (M + I)/2$ is again a fun exercise in formal algebra; as is confirming that we have discovered an amusing presentation of the symmetric group S_3.

Characteristic or minimal polynomials, entirely abstract for me as a student, now become members of a rapidly growing box of concrete symbolic tools, as do many matrix decomposition results, the use of Groebner bases, Risch's decision algorithm for when an elementary function has an elementary indefinite integral, and so on. Many algorithmic components of CAS are extraordinarily effective when two decades ago they were more like 'toys'. This is equally true of extreme-precision calculation — a prerequisite for much of my own work [2], [4] and others [5] — or in combinatorics. The number of *additive partitions* of $n, p(n)$, has ordinary generating function (o.g.f.) $\prod_{k=1}^{\infty}(1-q^k)^{-1}$. On a reasonable laptop calculating $p(200) =$ 3972999029388 naively from the o.g.f. took 20 minutes in 1991. Today it takes about 0.17 seconds while $p(2000) =$ 4720819175619413888601432406799959512200344166 takes two minutes naively and about 0.2 seconds using the built-in *Maple* recursion. Likewise, the record for computation of π has gone from under 30 million decimal digits in 1986 to over 1.6 trillion places this year.

Concluding remarks

We live in an information-rich, judgement-poor world and the explosion of information and tools is not going to diminish. We have to learn and teach judgement when it comes to using what is already possible digitally. This means mastering the sorts of tools I have illustrated. But it will never be the case that quasi-inductive mathematics supplants proof. We need to find a new equilibrium. The following identity

$$\sum_{n=-\infty}^{\infty} \operatorname{sinc}(n)\operatorname{sinc}(n/3)\operatorname{sinc}(n/5)\cdots\operatorname{sinc}(n/23)\operatorname{sinc}(n/29)$$
$$= \int_{-\infty}^{\infty} \operatorname{sinc}(x)\operatorname{sinc}(x/3)\operatorname{sinc}(x/5)\cdots\operatorname{sinc}(x/23)\operatorname{sinc}(x/29)\,\mathrm{d}x \qquad (9)$$

where the denumerators range over the odd primes was empirically discovered. Provably, the following is true: The analogous 'sum equals integral' identity remains valid for more than the first 10 176 primes but stops holding after some larger prime, and thereafter the 'sum less the integral' is positive but *much less than one part in a googolplex*. A stronger estimate is possible assuming the GRH [2].

That said, we are only beginning to scratch the surface of a very exciting set of tools for the enrichment of mathematics, not to mention the growing power of formal proof engines. I conclude with one of my favourite quotes from Jacques Hadamard [15]:

> The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.

Never have we had such a cornucopia of fine tools to generate intuition. The challenge is to learn how to harness them, how to develop and how to transmit the necessary theory and practice. The new Newcastle Priority Research Centre I direct, *CARMA*,[8] hopes to play a lead role in this endeavour.

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Puzzle corner

Norman Do[*]

Welcome to the Australian Mathematical Society *Gazette*'s Puzzle Corner. Each issue will include a handful of entertaining puzzles for adventurous readers to try. The puzzles cover a range of difficulties, come from a variety of topics, and require a minimum of mathematical prerequisites to be solved. And should you happen to be ingenious enough to solve one of them, then the first thing you should do is send your solution to us.

In each Puzzle Corner, the reader with the best submission will receive a book voucher to the value of $50, not to mention fame, glory and unlimited bragging rights! Entries are judged on the following criteria, in decreasing order of importance: accuracy, elegance, difficulty, and the number of correct solutions submitted. Please note that the judge's decision — that is, my decision — is absolutely final. Please e-mail solutions to ndo@math.mcgill.ca or send paper entries to: Gazette of the AustMS, Birgit Loch, Department of Mathematics and Computing, University of Southern Queensland, Toowoomba, Qld 4350, Australia.

The deadline for submission of solutions for Puzzle Corner 13 is 1 September 2009. The solutions to Puzzle Corner 13 will appear in Puzzle Corner 15 in the November 2009 issue of the *Gazette*.

Digital deduction

The numbers 2^{2009} and 5^{2009} are written on a piece of paper in decimal notation. How many digits are on this piece of paper?

Square, triangle and circle

Let $ABCD$ be a unit square and ABX an equilateral triangle with X outside the square. What is the radius of the circle passing through C, D and X?

Piles of stones

There are 25 stones sitting in a pile next to a blackboard. You are allowed to take a pile and divide it into two smaller piles of size a and b, but then you must write the number $a \times b$ on the blackboard. You continue to do this until you are left with 25 piles, each with one stone. What is the maximum possible sum of the numbers written on the blackboard?

Photo: Craig Jewell

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Do you know my number now?

Two geniuses are each assigned a positive integer and are told that the two numbers differ by 1. They then take turns to ask each other, 'Do you know my number now?'. If the geniuses always respond to questions truthfully, prove that one of them will eventually answer affirmatively.

Fun with floors

Prove the following interesting identity for every integer n greater than 1, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

$$\lfloor \sqrt{n} \rfloor + \lfloor \sqrt[3]{n} \rfloor + \cdots + \lfloor \sqrt[n]{n} \rfloor = \lfloor \log_2 n \rfloor + \lfloor \log_3 n \rfloor + \cdots + \lfloor \log_n n \rfloor$$

Noodling around

In front of you is a bowl containing 100 noodles. You randomly pick two ends of noodles and join them together until there are no more ends left and only noodle loops remaining.

(1) What is the probability that there is only one loop in the bowl?
(2) What is the probability that there are k loops in the bowl?
(3) What is the expected number of loops in the bowl?

Photo: James Rubio

Solutions to Puzzle Corner 11

The $50 book voucher for the best submission to Puzzle Corner 11 is awarded to Ivan Guo.

Bags and eggs

Solution by Ben Carr: The bag with the most eggs must contain at least nineteen of them. To see that the task is possible with nineteen eggs, place bag 1 and one egg into bag 2, then place bag 2 and one egg into bag 3, then place bag 3 and one egg into bag 4, and so on.

Area identity

Solution by Ivan Guo: Take the identity that we wish to prove and add $\text{Area}(AMP) + \text{Area}(CDR)$ to both sides to obtain the equivalent identity

$$\text{Area}(ABN) + \text{Area}(CDN) = \text{Area}(CMD).$$

This is in turn equivalent to the equation

$$\text{Area}(ABCD) - \text{Area}(AND) = \text{Area}(CMD),$$

which is true since $\text{Area}(AND) = \text{Area}(CMD) = \frac{1}{2}\text{Area}(ABCD)$. Note that the points M and N could have been anywhere on the sides AB and BC and the result would still hold.

Factorial fun

Solution by Laura McCormick and Rick Mabry: The square-free part of a positive integer x is the number $\frac{x}{n^2}$, where n^2 is the largest perfect square dividing x. We will write $x \equiv y$ whenever x and y have the same square-free parts. It is easy to see that this is an equivalence relation and we have the following.

$$\begin{aligned}\prod_{k=1}^{4n} k! &= 4n \times (4n-1)^2 \times (4n-2)^3 \times \cdots \times 3^{4n-2} \times 2^{4n-1} \times 1^{4n} \\ &\equiv 4n \times (4n-2) \times \cdots \times 4 \times 2 = 2^{2n}(2n)! \equiv (2n)!\end{aligned}$$

This means that, for every positive integer n, the number

$$\frac{1}{(2n)!}\prod_{k=1}^{4n} k!$$

is a perfect square. For our particular problem $n = 25$ and hence, we must erase 50! so that the product of the remaining 99 numbers is a perfect square.

In fact, Laura and Rick managed to solve the far more general problem of determining for which positive integers $m \leq n$ the following number is a perfect square.

$$\frac{1}{m!}\prod_{k=1}^{n} k!$$

Highway construction

Solution by Stephen Muirhead: For the sake of contradiction, assume that the highway never reaches 100 kilometres in length. Let X_k denote the length of highway, in kilometres, built in the first k months of construction. Then $X_k < 100$, so we have $\frac{1}{X_k^{100}} > \frac{1}{100^{100}}$ for all k. In other words, more than $\frac{1}{100^{100}}$ kilometres of highway are built every month. Thus, after 100^{101} months, at least 100 kilometres of highway have been built in total, a contradiction.

Busy bee

Solution by Ivan Guo: Suppose that the bee starts at the point A and, after flying for two metres, is at the point B. Let the midpoint of AB be O and consider the sphere of radius one metre with centre O. If the bee visited the point P outside the sphere during its flight from A to B, then let P' be the point such that O is the midpoint of PP'. Note that $APBP'$ is a parallelogram with O as its centre. Since the shortest path between two points is a line segment, the distance the bee flew between A and B is at least $AP + PB = P'B + PB \geq PP' > 2$. But this contradicts the fact that the bee flew two metres from A to B. The same argument shows that the bee could not have escaped the sphere on its return flight from B to A.

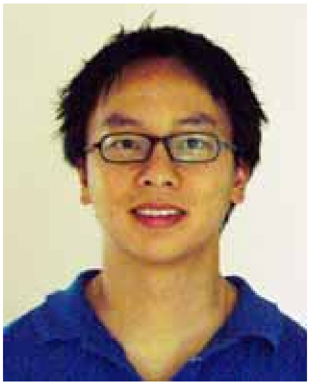
Coin-flipping games

Solution by Jamie Simpson:

(1) Let the two players each toss the coin once, with a player winning if they obtain heads while their opponent obtains tails. In the event that the outcomes are the same, simply repeat the procedure until the outcomes differ. Clearly, no player has an advantage in this game so each player has probaility $\frac{1}{2}$ of winning.

(2) Let the two players take turns tossing the coin, with a player winning if they obtain the first head. Then the first player's probability of winning is $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \cdots = \frac{2}{3}$ and the second player's probaility of winning is $\frac{1}{3}$.

(3) Let the binary expansion of $\frac{1}{\pi}$ be $0.b_1b_2b_3\ldots$. Let the two players take turns tossing the coin until a head appears and suppose this happens on toss k. If $b_k = 1$, we say that the first player wins and if $b_k = 0$, then we say that the second player wins. Then the first player's probability of winning is $\frac{b_1}{2} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \cdots = \frac{1}{\pi}$.



Norman Do is currently a CRM-ISM Postdoctoral Fellow at McGill University in Montreal. He is an avid solver, collector and distributor of mathematical puzzles. When not playing with puzzles, Norman performs research in geometry and topology, with a particular focus on moduli spaces of curves.

Mathematical minds

Robert McIntosh*

Gazette: Where did you grow up?

McIntosh: My father, Gaius McIntosh, was the first lecturer in Philosophy at the University of New England (UNE), Armidale, northern NSW, moving there in 1947 (then an offshoot of Sydney University). My mother, Edna McIntosh, was also a great believer in education, and she and Dad raised seven children, all of us gaining degrees from UNE. The seven of us are Alan (who became head of Mathematics at ANU), Colin (who became head of Mathematics at Monash), Graeme (a head mathematics teacher), Malcolm, Dorothy, Cathy and myself (the youngest). It was said that my dad took up Philosophy because he found mathematics too easy! At an early age we played games my father devised that involved mathematics.

Gazette: Did you enjoy maths at school?

McIntosh: When I was in grade two primary at Armidale Public School I remember enjoying a mathematics book that had lists of square numbers, log tables, formulas et cetera. I remember learning many square numbers off by heart, and one that caught my imagination was 13 456. My teacher, Mrs Bozzer, asked the class to use four mathematical operations to end up with 10. She was expecting answers like $4\times2-1+3=10$. I gave the answer $\sqrt{13\,456}-10^2-4^2+\sqrt{100}=10$, or something like that, and Mrs Bozzer had to check with the head-mistress if I was correct. My brother Graeme recently told me that this response was still being talked about at a NSW Department of Education conference as recently as a few years ago (around 35 years later!). A book I enjoyed when I was 13 was *Cheaper by the Dozen* (a 1948 book written by Frank Gilbreth Jr and Ernestine Gilbreth Carey) where the father taught his children to answer questions like 43×37 quickly (to impress the headmaster) by using the trick that $43\times37=(40+3)\times(40-3)=40^2-3^2$, which is easy to calculate. At the Armidale High School, I was very fortunate to have Pop Farrell and John Galvin as my mathematics teachers, as they were both supportive and able to create an enjoyable environment. Pop Farrell would say that 80% is a 'pass mark', as a score above 80% shows that you have understanding of the material.

Gazette: What did you want to do when you started university? What made you go on to do a PhD in maths?

McIntosh: University was an environment that I really enjoyed, mostly due to its freedom and the need for the student to be self-disciplined if they are to succeed. I question the pressure that society places on our Year 12 high-school students (some of it driven by a school's financial needs) when people excel at different

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times in their lives and it is perhaps the higher level of education that we should be emphasising. My enjoyment and success in mathematics increased significantly at UNE (gaining first class honours) with supportive lecturers like Mike Campfell, Norm Dancer, Joe Hempel, a terrific tutor Simon Smith, and fellow students Ian Roberts and Mirka Miller who had a great love of maths. UNE was such a good environment for learning mathematics.

My love of mathematics was driven by its ability to help us understand our physical world and subjects like cosmology enthralled me. Wes Taylor was my encouraging Honours supervisor in General Relativity. That love of learning about physical systems encouraged me to undertake a PhD at ANU. It was an environment with many top international mathematicians, and took me to Stanford University in California, where I met my wife, Wendy.

My career in industry may have been better served if I had undertaken a subject such as electrical engineering but I feel that mathematics is a very enriching subject. A mathematical background often provides me with the confidence and skills to tackle problems that others may not.

Gazette: What made you decide to work in industry? Where have you worked?

McIntosh: After coming from a family heavily involved in the academic environment, the world of industry was novel and interested me. I was at BHP Research for 10 years (becoming a Senior Research Scientist) and Telstra Research Laboratories for seven years, until both were closed. I am now with Telstra's Chief Technology Office and am currently one of a fortunate few to be doing research (in radiation safety). This is an area that involves a broad range of skills in physics, statistics, biology, and electrical engineering as well as maths. We are part of the collaborative effort at the Australian Centre for Radiofrequency Bioeffects Research. Most of the mathematical work is performed using computational packages (that solve Maxwell's electromagnetic equations) and I often long for the time when computers weren't around. Recently I coded up a finite-difference solver for thermal analysis which can calculate the induced heat in a human body due to radiofrequency (RF) radiation.

My most inspiring boss was a man called Dr Peter Ellis (when I was at BHP). Peter was the stereotypical absent-minded professor type and there were many stories about Peter (including stories told by himself) about his absent-minded ways. He would often come in to work and tell us about ideas that he had at 4 a.m. and the team would then go about trying to implement his ideas. Those days were very exciting and we would design many devices to control the flow of liquid metal. We used powerful six Tesla permanent magnets to break the flow of molten steel being cast, electrical coils that would levitate liquid Zincalume as steel strip was being coated, and highly efficient non-contact pumps (designed with permanent magnets) that would pump the highly corrosive Zincalume. Maths in its many forms was used in the design process. Checking and matching theory with physical testing is always key to good understanding and design.

Another area that I worked where maths was important at BHP was in signal processing for airborne electromagnetic exploration. Noise reduction methods were sought to provide better quality maps of the geological structures.

Gazette: What is the best career advice you have ever received?

McIntosh: Probably lots but none that I can specifically remember! I guess if I was to give advice it would be to work in areas that you are passionate about.

Gazette: What achievement are you most proud of?

McIntosh: I am very fortunate at the moment to be able to supervise university students (as an industry supervisor) and that is a great pleasure. One PhD student, Teddy Kurniawan, looked at analytical methods to solve for the electromagnetic field in the near field of an RF source. On a personal note, I have twice won an award for best presentation at a conference — the B.H. Neumann student award (Bernard was such a nice man) at the Australian Mathematical Society conference at UWA in 1986; and the best paper award at the Australasian Radiation Protection Society Conference in Sydney in 2006. The key for speaking in public is to prepare thoroughly and to give much thought to the material. My youngest daughter, Whitney, recently won a BHP award for mathematics and the four mathematicians that made the presentations at the awards ceremony gave poor talks. If mathematics is to be promoted then we must take every opportunity to connect with the audience and our youth need to see why mathematics is relevant, including how it can be used to give insight to our physical world.

Rob McIntosh obtained his PhD in Mathematics at the Australian National University in 1989 in the area of PDEs. He has been a member of the Electromagnetic Energy Safety Research team at Telstra since 1999, developing and applying a numerical modelling environment for the study of radiofrequency dosimetry and human body absorption. Rob is also a member of the Australian Centre for Radiofrequency Bioeffects Research and an Adjunct Professor at Swinburne University of Technology. Between 1989 and 1999, he worked at the BHP Research Laboratories on the development of electromagnetic levitation, pumping, and braking devices for liquid metal, and new techniques in noise reduction in electromagnetic geophysics. Rob is a member of the Australian Mathematical Society. (Photo by Leah McIntosh.)

AG seminars: protocols and using VLC media player

Bill Blyth*

Introduction

There are many competing video-conferencing systems available and the use of video conferencing will increase. It is reassuring that developments in video conferencing confirm that the mathematical communities in Australia, New Zealand, Canada and the UK have made sound choices to use Access Grid (AG) technology.

As end users, our main issues now centre around our local AG IT support and the support of network engineers (specifically with respect to multi-casting). The Australian Research Collaboration Services (ARCS) has chosen two systems to support: AG for large rooms and high bandwidth and EVO[1] which is ideal for desktops and lower bandwidth internet connections. Usually, different video-conferencing systems do not communicate with each other. However, a bridge for EVO to AG is in the final phase of testing by ARCS and is expected to be available very soon. When available to the Australian community, there will be an announcement on the ARCS website (http://www.arcs.org.au). This would mean that a researcher at home or office (without the high bandwidth needed for AG) could participate in an AG session. There is now a clear advantage for the mathematical community to adopt EVO for limited video-conferencing collaboration (such as small meetings for planning, or PhD supervision).

The national program of collaborative teaching of advanced mathematics at Honours level at multiple remote sites by using the AG is now established. Although some seminars have been offered via the AG, we will soon have a seminar series established including some high-profile seminars.

Seminars on optimisation

An Optimisation Group has recently been formed as a Special Interest Group of ANZIAM and is planning a comprehensive list of activities in the area of Optimisation. Planning meetings are being held weekly via the AG and the Optimisation Group has agreed to organise and conduct 'The AMSI Optimisation AGR Colloquium Series'. The leaders are Jonathan Borwein (Newcastle), Andrew Eberhard (RMIT) and Regina Burachik (UniSA). They are all experienced with the use of the AG. Protocols for AG sessions and seminars are being developed and tested (the author is assisting with this and has also arranged for expert AG IT advice from Jason Bell of ARCS). 'Test' and 'trial' seminars are already being conducted across the three lead Access Grid Rooms (AGRs). This testing phase is nearly

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[1]EVO is a PC-based multipoint collaboration tool which features advanced video-conferencing.

completed and so the seminars will soon be available to others. The Optimisation Group has agreed that any AGR wishing to participate in their activities will be required to have completed the Quality Assurance process. This will help to ensure that AGRs are correctly set up. Given the expertise and strong focus of the Optimisation Group, the planning and organisation of the AGR Colloquium Series should provide much valuable information for other groups to follow. AMSI's main role here is to facilitate collaboration via the AG and to offer web hosting.

AG seminars by Terry Tao

Terry Tao, Fields Medalist, is a Clay-Mahler Lecturer in 2009 and there are plans for him to give some presentations over the AG. Andrew Hassell, at ANU, is coordinating these lectures and will soon be advertising the details widely. AMSI is providing advice. It's expected that the AG lectures will be:

- discrete random matrices, hosted from Monash University, Wednesday 2 September, early- to mid-afternoon;
- compressed sensing, hosted from a large lecture theatre at UWA, equipped as an AGR, Friday 4 September, 1–2 pm;
- TBA, hosted from the Baume AGR, ANU, Wednesday 23 September, 3:30–4:45 pm.

Appropriate protocols for the presentation, and for the AGRs that wish to join, are currently being developed.

AG seminars: protocols

The development of protocols is a work in progress as we draw from our AMSI experience (with AGR Honours teaching and seminars), the AGR IT community (particularly Jason Bell), the Canadian experience (since 2005) with their coast-to-coast seminars, C2C [3] and the experience and experimentation of the newly formed Optimisation Group.

The protocols will include the following elements.

- Quality assurance (QA) of AGRs. This is a process being taken up internationally and is being driven by Jason Bell. The AGR IT people make an appointment (email j.bell@cqu.edu.au) for about an hour to have their AGR setup checked. Once QA-ed, the AGR may join in activities such as the AG Clay-Mahler Lectures and the Optimisation seminars.
- VPCScreen[2] (and AGVCR) [2]. Most presentations (for Honours lectures and seminars) do not require control of the software to be given to remote nodes. Although VNC is being used, VPCScreen should be used instead: because VPCScreen produces a video stream of the presentation material, it scales up to a large number of AGRs. It also can be recorded by AGVCR.

[2]See *Gaz. Aust. Math. Soc.* 36(3), 105–109 for more information about VPCScreen (http://www.austms.org.au/Publ/Gazette/2009/May09/AccessGrid.pdf).

- VenueVNC [2]. This should be used if remote control of the software is required (for Honours interactive tutorials and interactive research). This works well only for a small number (about five to eight) of nodes. It is possible to use 'multicast' versions of VNC, such as used in the C2C seminar series [3] in Canada (where VNC Reflector is used).

  A mix of VPCScreen and VenueVNC was used by Jason Bell to run the one-day workshop on mathematics for 21st century engineers hosted by RMIT with 16 remote AGRs participating. The five nodes presenting used VNC and all others used VPCScreen, see Figure 1 for a 'wall' screenshot while the presentation was made from Monash. For such an event, expert AG IT support is necessary.

Figure 1. A screenshot of the 'wall', where a mix of VPCScreen at receiving AGRs and VenueVNC at presenting AGRs was used in December 2007 by Jason Bell to run the one-day workshop on mathematics for 21st century engineers.

- VLC Media Player. A presentation that requires a video (with audio) to be played presents a challenge for the AG. VPCScreen [2] handles video (without audio) and animations, such as Maple animations, but doesn't handle videos such as mpeg and mov files. VLC Media Player is open source, cross platform and handles many video formats.

  RMIT (with UniSA and W'gong AGRs) successfully tested the use of VLC to stream some video files. Mark Nelson (from W'gong) gave a seminar at RMIT to La Trobe that discussed the use of videos in mathematics teaching. Mark had many videos to show: the first was successfully shown but then the VLC was accidentally exited! Although we could reinstate VLC, we were unable to restart the web streaming: the player is simple to use, but the web streaming requires expert IT support to set up. This leads to our next two items.

- Specialist AG IT support. It's important that specialist AG IT support staff are available for testing of current hardware and software and maintenance. They should be present throughout any special events and seminars. This should not be necessary once an Honours course is established since the Lecturer is usually the same throughout and the remote sites that are connecting remain unchanged.
- Data Storage (see p. 12 of [1]). The Access Grid Toolkit (AGTk) allows users to upload and download files to the Venue Server. This allows those connected via AGRs to have access to various collaborative files or presentation materials. We recommend that presentations be provided at least two days before the presentation, so that they can be tested at the host AGR, placed into the data storage and then downloaded as a local copy at each of the remote AGRs.

  Providing files for testing and copying into the Data Storage gives a backup capability: for example, mpeg videos (which are tricky to stream properly) would be available at each AGR where playing locally using VLC would be a simple task.
- List of software and versions. For example, we might recommend that all AGRs have Adobe Reader versions 8 and 9 (the two most current versions) and pdf files must be compatible with these. This would have avoided difficulties with a recent pdf presentation where one of the AGRs had Adobe Reader version 4 and could not read the more recent file!

Conclusion

Collaborative teaching of advanced mathematics across Australia via the Access Grid is expanding with the participation of New Zealand. National seminars are also offered over the AG, but specialist seminar series and some of the Clay-Mahler Lectures will be offered very soon. Protocols are being developed and tested so that the seminars proceed with a minimum of problems.

Acknowledgements

The author's position at AMSI is, in part, supported by the *Collaboration and Structural Reform* project, nGame, funded by the Australian Government and based at the University of Sydney. The author thanks Jason Bell (at CQU and an Access Grid specialist within the Australian Research Collaboration Services); the leaders of the Optimisation group: Jon Borwein, Andrew Eberhard and Regina Burachik (and their AG IT support staff); and Andrew Hassel and others who are organising the selected Clay-Mahler Lectures given by Terry Tao via AGRs.

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Bill Blyth is Adjunct Professor of computational mathematics at RMIT University and was Head of the Department of Mathematics for $6\frac{1}{2}$ years. He is Chair of the Engineering Mathematics Group of Australia, a Center Affiliate at the International Centre for Classroom Research (at the University of Melbourne) and led the design, construction and initial delivery phases of the RMIT University AGR. He is currently at The Australian Mathematical Sciences Institute, AMSI, as the national coordinator of AMSI's AGR project. His PhD was in theoretical physics at Imperial College, London. He has an unusually broad range of research interests in mathematics education (in technology-rich classrooms) and the numerical solution of differential and integral equations. He has published more than 60 refereed papers.

Communications

The state of mathematical sciences in Australia

Hyam Rubinstein*

Mathematics and statistics in Australia face substantial challenges. The recent Federal Budget and the important Bradley review of the higher education system in Australia barely mention the mathematical sciences. On the other hand, the National Strategic Review of Mathematical Sciences[1], released in 2006, identified a serious decline in mathematics education in both schools and universities. The 2009 Budget contained a number of measures to improve higher education, including ambitious targets for equity and access. It would seem that the problems in mathematical sciences will be a substantial obstacle to achieving these targets, especially in key areas such as engineering, economics and commerce, environment and climate change, bioinformatics and biostatistics, resource allocation and infrastructure. It is important to note that the mathematical sciences community has been producing evidence of the problems, strategic plans and submissions to numerous reviews for the last three years, with little effect, except requests for more submissions, plans and evidence!

Firstly, the problems[2]. To summarise, between 1995 and 2007, the number of Australian Year 12 students doing advanced mathematics dropped from around 25 000 to around 20 000. Moreover, in the international study of comparative performance 'Trends in International Mathematics and Science Study', Year 8 Australian students performance declined from above average to below average, in the period 1995–2007. Australia is now behind both the US and UK, whereas we were ahead of these countries in 1995. In 2007, 40% of senior mathematics teachers did not have a three-year university degree in mathematics — this has risen from 30% in 1999. Finally, perhaps the most worrying trend is that Australia graduates around 40% of the OECD average for university majors in mathematics and statistics, scaled for our population size. In 2001 there were around 2100 such graduates whereas in 2007 this had declined to 1800. Government figures show an increase in demand by the Australian economy of 52% for mathematicians and statisticians between 1998 and 2005. A related figure is that the number of academic positions in university Mathematics and Statistics Departments has declined by approximately 40% since 1995.

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This article first appeared on *The Funnel Web* at http://www.the-funneled-web.com in June 2009.

[1] http://www.review.ms.unimelb.edu.au/Report.html
[2] For the full story, see www.amsi.org.au/pdfs/National_Maths_Strategy.pdf

Certainly the federal government has been focusing on big issues in the economy, climate change, industrial relations, but has also promised an education revolution. Resources have been put into computers in schools, new school and university buildings and super science projects. The main budget initiative, which is relevant to the above problems, is HECS reductions for students studying science and mathematics, who become teachers in schools. Although this is a commendable scheme, it may do little to address the serious shortage of mathematics teachers.

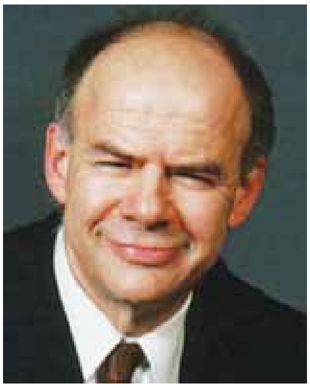
Firstly, Australia already produces more than 25 science graduates for every mathematics or statistics student. So the most likely outcome will be more science students going into teaching, whereas the opportunities for the very small numbers of mathematical sciences students are much better in industry than teaching. Secondly, most university science courses have few requirements for studying mathematics or statistics and the registration procedures for teachers, in terms of subject knowledge, vary greatly between different states in Australia. So we may well end up with underqualified science students teaching mathematics in schools.

All is not doom and gloom though. Australia produces marvellous talents in the mathematical sciences — Berkeley, Stanford, Caltech, UCLA, Chicago, MIT, Cambridge et cetera, have many outstanding mathematicians and statisticians who were educated in Australia. Invitations to speak at prestigious international conferences and being on editorial boards of top journals are common for Australian mathematical scientists. Australia has a wonderful tradition of excellence in mathematics competitions in schools, run by a small army of dedicated teachers and academics and coordinated by the Australian Mathematical Trust, with very little government support.

Finally the mathematical sciences community has banded together to set up the Australian Mathematical Sciences Institute, which recently won a National Innovation Award for its program of industry internships for mathematical sciences students. AMSI has been active in school education, producing an excellent series of modern mathematics textbooks suitable for schools across Australia, and fostering research and interaction with industry. AMSI is a model of the hub and spokes concept, coordinating national activities such as advanced summer courses in areas of interest for honours and graduate students, assisting with running focused workshops (Future models for Energy and Water Management is to be run in July in conjunction with UNESCO), but not yet recognised by the federal government.

Why are we struggling to gain attention? The resources involved are very small indeed — our budget request was less than 3% of the super science projects. I believe there are two reasons. Firstly, there is no 'mathematics industry' to lobby the government to take action. On the other hand, every industry needs and uses mathematics and statistics. Every time a company does a market survey, performs risk analysis, tries to make logistics more efficient, et cetera, they are using skills from the mathematical sciences. Computing power and software do not replace this; for example the advances in scheduling enabling very complex tasks like running airlines, relies on advances in mathematical algorithms much more than progress in computing speed.

Secondly, mathematical sciences are not science, in the sense that we use no equipment other than brain-power and computer-power. So we have no large impressive facilities that politicians can take credit for. But mathematics and statistics are becoming more and more important as tools to tackle the problems of society, from efficient use of infrastructure such as ports and airports, to climate change to the genetic revolution in medicine. If Australia continues to neglect its basic skills in the mathematical sciences, turning this around will become more and more difficult. To quote the international reviewers from 2006: '... we found the nation's distinguished tradition and capability in mathematics and statistics to be on a truly perilous path'.



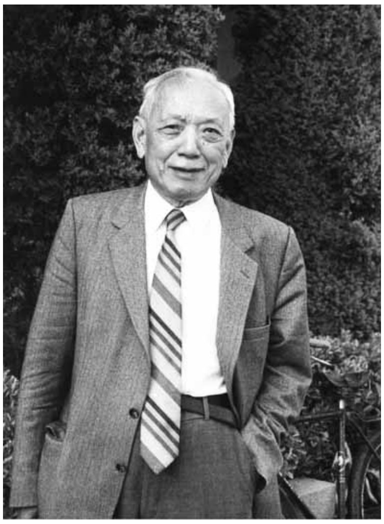
Hyam is Chair of the National Committee for Mathematical Sciences and was the Chair of the working party of the National Strategic Review of Mathematical Sciences Research which was completed during 2006. He is interested in geometric topology, differential geometry, shortest networks and has been at Melbourne University so long that he gets to walk behind the Chancellor at academic processions for graduation ceremonies.

Chern Medal Award
New prize in science promotes mathematics

Professor Dr Martin Grötschel*

The International Mathematical Union (IMU) and the Chern Medal Foundation (CMF) are launching a major new prize in mathematics: the Chern Medal Award. The Award is established in memory of the outstanding mathematician Shiing-Shen Chern (1911, Jiaxing, China – 2004, Tianjin, China).

Professor Chern devoted his life to mathematics, both in active research and education, and in nurturing the field whenever the opportunity arose. He obtained fundamental results in all the major aspects of modern geometry and founded the area of global differential geometry. Chern exhibited keen aesthetic tastes in his selection of problems, and the breadth of his work deepened the connections of modern geometry with different areas of mathematics.



Professor Shiing-Shen Chern

The Medal is to be awarded to an individual whose lifelong outstanding achievements in the field of mathematics warrant the highest level of recognition. The Award consists of a medal and a monetary award of US$500 000. There is a requirement that half of the award shall be donated to organisations of the recipient's choice to support research, education, outreach or other activities to promote mathematics. Professor Chern was generous during his lifetime in his personal

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support of the field and it is hoped that this philanthropy requirement for the promotion of mathematics will set the stage and the standard for mathematicians to continue this generosity on a personal level. The laureate will be chosen by a Prize Committee appointed by the IMU and the CMF.

The IMU has been awarding the Fields Medals — generally considered the 'Nobel Prize for mathematics' — since 1936, the Nevanlinna Prize in the field of theoretical computer science since 1982, and the Gauss Prize for applied mathematical work since 2006. The Fields Medals and the Nevanlinna Prize are given to young researchers below 40 years of age, in recognition of specific path breaking results. The Gauss Prize recognises mathematical results that have opened new areas of practical applications.

With the Chern Medal Award, the IMU is broadening the range of its awards, now including long-term work with outstanding theoretical consequences.

The Nevanlinna Prize, Gauss Prize, and up to four Fields Medals are awarded every four years at the opening ceremony of the International Congress of Mathematicians. The Chern Medal Award will be awarded in the same manner, for the first time at ICM 2010 in Hyderabad, India on 19 August 2010. The award ceremony will include an overview of the achievements of the prize-winner. The presentation of the mathematical work will be addressed to the general public as well as journalists, so that all may appreciate the vitality of mathematics in our times.

Chern Medal, program guidelines

I. Selection of Medalists. The Medal is to be awarded to an individual whose accomplishments warrant the highest level of recognition for outstanding achievements in the field of mathematics.

All living, natural persons, regardless of age or vocation, shall be eligible for the Medal. Additional qualifying or disqualifying criteria may be imposed consistent with IMU policies.

II. Selection Committee. Each Medalist shall be selected by a Selection Committee consisting of five members.

A new Selection Committee shall be formed at least two years in advance of each award ceremony, for the purpose of identifying that Medalist.

The IMU shall appoint four members of each Selection Committee, and CMF shall appoint the fifth.

At the outset of each award cycle, the IMU shall nominate one of the five members of the Selection Committee as Chair of the Committee, subject to approval by CMF.

III. The Awards. A. Each Medalist shall receive a cash prize of \$250 000.

B. In addition, each Medalist may nominate one or more organisations to receive funding totaling \$250 000, for the support of research, education or other outreach programs in the field of mathematics (the 'Organisation Awards').

C. Each Medalist's nominees for Organisational Awards are subject to approval by FIMU, which shall grant such approval so long as the proposed awards:

- will be qualifying, charitable distributions within the meaning of US tax law; and
- will substantially further the purposes of CMF, FIMU and the Grant.

IV. Other terms and conditions. The identity of each Medalist will be kept strictly confidential until the time at which that Medalist is honoured.

CMF retains the right to enjoin any ancillary usage of any name or mark associated with the Program if, in CMF's sole discretion, such use does not appropriately reflect on or advance the interests of the Program.

Nominations for the Chern Medal Award

The International Mathematical Union (IMU) and the Chern Medal Foundation (CMF) have, following the guidelines above, appointed a Selection Committee consisting of five members. This committee is chaired by the former IMU Secretary and the Past Director of the Institute for Advanced Study in Princeton Phillip A. Griffiths.

Nominations should ideally be sent by 15 December 2009 to the following address electronically or on paper:

Phillip A. Griffiths
Institute for Advanced Study Einstein Drive Princeton, N.J. 08540 USA.
E-mail: pg@ias.edu

Nominations must include:

- name and affiliation of candidate;
- a description of the work that qualifies the candidate for the award, written in terms that are accessible to mathematicians of different backgrounds, including references to the candidate's important papers;
- nominations are confidential, and must not be disclosed to the candidate.

SIAM names Fellows for key contributions to applied mathematics and computational science

Jessica Stephenson*

The Society for Industrial and Applied Mathematics (SIAM) has announced the SIAM Fellows Class of 2009 and the inauguration of the SIAM Fellows Program. Fellowship is an honorific designation conferred on members distinguished for their outstanding contributions to the fields of applied mathematics and computational science. During this inaugural year of the program, SIAM will confer Fellows status on 183 noteworthy professionals who will be recognised during the 2009 SIAM Annual Meeting in Denver, Colorado.

Seven Australian mathematicians are among those honoured with this award in 2009: Michael Barber, Richard Brent, Tony Guttmann, David Hill, Peter Kloeden, Michael Osborne and Ian Sloan. SIAM President Douglas N. Arnold said that the announcement of the first class of SIAM Fellows was an important milestone for the applied mathematics and computational science community. He continued:

> Reflecting the diversity of the SIAM membership, these men and women come from five continents, and work in academia, industry, and government laboratories. Advancing the frontiers of research in branches of mathematics as distinct as number theory and partial differential equations, these professionals have applied their work to endeavors ranging from mining to medicine. They have designed algorithms to make computing possible and written textbooks to train the next generation of mathematicians. Their contributions are truly outstanding.

The goals of the SIAM Fellows Program are:

- to honour SIAM members who are recognised by their peers as distinguished for their contributions to the discipline;
- to help make outstanding SIAM members more competitive for awards and honours when they are being compared with colleagues from other disciplines;
- to support the advancement of SIAM members to leadership positions in their own institutions and in the broader society.

Following 2009, the anticipated number of fellowships conferred annually will be approximately 0.3 percent of the number of regular SIAM members. For additional information about the SIAM Fellows Program and for nomination information, see http://www.siam.org/prizes/fellows/index.php.

Newly announced 2009 SIAM Fellows and their affiliations at the time of designation can be found at http://fellows.siam.org.

Congratulations to the seven recipients on this outstanding achievement.

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Report on ICIAM board meeting Oslo, 21 May 2009

Neville de Mestre*

I travelled to Oslo in the week 17–23 May to attend the ICIAM board meeting on Thursday 21 May as well as the Abel and Ramanujan prize ceremonies on 19 and 20 May.

The Abel prize was awarded to Mikhail Gromov, a Russian who has lived in France for many years and made great contributions to modern global Riemannian geometry. The prize was awarded by King Harald of Norway.

The Ramanujan prize for the best young researcher from a developing country was awarded to Enrique Pujal from Brazil.

The ICIAM board meeting went from 9.00am to 6.10 pm with a short break for lunch. It was held at the Norwegian Academy of Science and Letters.

Many matters were raised during the day and those of interest to Australia and New Zealand are listed below.

(1) The possible venues for ICM2014 are Brazil, Canada or South Korea. One of the candidates tried to solicit ICIAM's support for its bid, but it was decided not to show any preference.
(2) The current financial situation is that ICIAM has approximately US$85 000 in its bank account.
(3) Barbara Keyfitz from Ohio state was elected unopposed as President-Elect to assume the role in October 2009. She will become President in 2011. Ian Sloan will cease being Past-President in October, but has indicated that he is prepared to stand for ANZIAM rep in 2010. I will stand down at the AGM in February so that Ian and others can contest an election.
(4) Arvind Gupta, the Director of ICIAM 2011 in Vancouver, gave an update on progress. They are expecting between 2500 and 3500 delegates. The board agreed to set aside $17 000 for the Congress organisers to use exclusively for delegates from developing countries to attend. Although Australia and New Zealand are not developing countries, ANZIAM members are encouraged to nominate mathematicians whom they know from New Guinea, Indonesia or the Pacific Islands to apply to the Congress Director for a grant.
(5) The 27 mathematicians nominated by the Scientific Program Committee for the 2011 Congress were approved by the board.
(6) The venue for the 2015 congress will be Beijing. The Chinese application defeated the Japanese application by 13 votes to 10.

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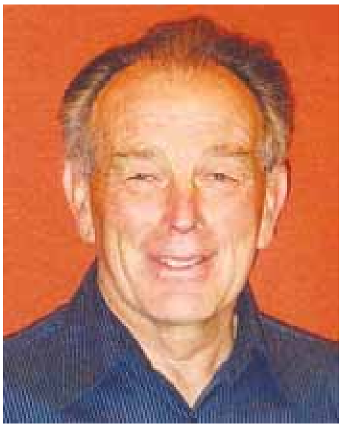
(7) Because Barbara Keyfitz is the current Treasurer of ICIAM, there will be a need to find a temporary treasurer at the next board meeting in 2010. Barbara will act in this position until a new treasurer is found. The position can be handled mainly by email, with the accounts being channelled through the SIAM administration in Philadelphia. The board meets once each year, when the Treasurer would be expected to attend, with the Treasurer's travel and accommodation met by ICIAM. If any member of ANZIAM is interested in becoming the temporary treasurer from October 2010 until September 2011, they should contact me for information on how to apply.
(8) The IMU representative attended the meeting as an observer. He announced a new mathematics prize, the Chern Prize, worth US$500 000 in any area of mathematics. Check the IMU website shortly for more information.
(9) It was decided that the five winners of 2011 ICIAM prizes would be invited to give a 30-minute talk on their research at an appropriate time slot in the Vancouver program. Nominations for these ICIAM prizes close on 21 September 2009.
(10) There was much discussion about the rules for conflict of interest when a member of the judging panel for a prize has performed joint work or is closely associated with a candidate.
(11) The board approved $3500 each to be given to conference organisers from both Brazil and Colombia to help researchers from developing countries attend the conferences in 2010 in these countries.
(12) The next board meeting is in Delhi, India, in August 2010.



Neville de Mestre retired from Bond University in 2004 as Emeritus Professor of Mathematics. Neville lectured at RMC Duntroon and ADFA from 1962 to 1989. He moved to Bond University as soon as it opened in 1989. In 1992 he inaugurated the Biennial Australasian Mathematics and Computers in Sport conferences, which eventually became the Mathsport Special Interest Group of ANZIAM. Neville has directed or co-directed seven of the first nine conferences. Neville has also been Chair of ANZIAM (during ICIAM 2003 in Sydney) and Deputy Chair on a number of occassions. He was director of ANZIAM conferences at Smiggin Holes (1971), Merimbula (1984) and Coolangatta (1998). His research interests include fluid mechanics, bushfires, sport and mathematical education at all levels. He founded the ACT Mathematics Centre which now travels Australia through Questacon. He gave the tenth G.S. Watson Memorial Lecture at La Trobe University (Bendigo) in 2008.

On the enumeration of Pythagorean triples having a fixed base length

M.A. Nyblom*

Abstract

An enumeration formula for counting the number of partitions of an integer $n > 1$ having parts in arithmetic progression of common difference two is derived in terms of the number of divisors of n. As a consequence, a formula is obtained giving the number of Pythagorean triples in which n occurs as one of the base lengths in terms of the number of odd and even divisors of n^2.

Introduction

Among the many results of classical number theory attributable to Fermat, one of the most well known asserts that any prime $p \equiv 1 (\bmod 4)$ is always representable as a sum of two squares. This result was later extended by Euler who proved that an integer n is representable as a sum of two squares if and only if when n is expressed as a product of prime powers, every prime factor $p \equiv 3 (\bmod 4)$ occurs with an even exponent. However, it was Jacobi who showed that one could count the number of such representations of n via an enumeration formula, given in terms of positive divisor functions. Indeed, if for each $i \in \{1, 3\}$, one defines $D_i(n)$ as the number of positive divisors d of n such that $d \equiv i (\bmod 4)$, then $r_2(n)$, the number of representations of n as a sum of two squares, is given by

$$r_2(n) = 4(D_1(n) - D_3(n)). \tag{1}$$

(Noting here that the presence of the multiplicative factor of 4 in (1), is to take into account the four possible combinations on the signs of the pair of numbers being squared.) By considering only the sum of squares of positive integers, we see one immediate consequence of the Jacobi formula is that $r_2(n^2)/4 - 1$ counts the number of right-angle triangles with integer side lengths, in which a positive integer n appears as a hypotenuse length. To illustrate take $n = 5$, then as $D_1(n^2) = 3$, $D_3(n^2) = 0$, we have $r_2(n^2) = 12$ and so $r_2(n^2)/4 - 1 = 2$, which corresponds to the two right-angle triangles represented by the triples $(3, 4, 5)$ and $(4, 3, 5)$. Recalling that a Pythagorean triple is a 3-tuple of integers (x, y, z), with $0 < x < y < z$ and satisfying the equation $x^2 + y^2 = z^2$, one may, in view of the previous observation, question whether it is possible to find a similar enumeration formula, in terms of positive divisor functions, for the number of Pythagorean

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triples in which n appears as one of the base lengths. In this paper, we show that such a formula does indeed exist, and moreover is given by

$$\tfrac{1}{2}(d_0(n^2) + (-1)^{n+1} d_1(n^2) - 1), \tag{2}$$

where, for each $i \in \{0, 1\}$, one defines $d_i(n)$ as the number of positive divisors d of n such that $d \equiv i (\bmod 2)$. It is of interest to note that the expression in (2) can also be written in terms of the simple divisor function $d(n)$, since using the relation $d_1(n) = d(n) - d_0(n)$ and noting $d_0(n) = d(n/2)$ if n is even and 0 otherwise, we find that (2) reduces to the function $f(n)$ where

$$f(n) = \begin{cases} \dfrac{d(n^2) - 1}{2} & \text{for } n \text{ odd}, \\ \dfrac{2d(n^2/2) - d(n^2) - 1}{2} & \text{for } n \text{ even}. \end{cases}$$

The expression in (2) will be derived from a curious enumeration formula which counts the number of partitions of an integer having parts in arithmetic progression of common difference two. We begin in the next section with a small digression on the construction of this partition formula.

Main results

It is well known that the number of consecutive integer partitions of a positive integer n, that is partitions having consecutive integer parts, is one less than the number of odd divisors of n. Indeed, this particular result has rather a long history, as it first appeared as a problem of LeVeque in [3] and has subsequently reappeared as a proven theorem in [2] and [4]. Although there exists a criterion for n to be decomposable into a partition having parts in arithmetic progression with a prescribed common difference (see [1]), it appears that no investigation has been made into the construction of enumeration formulae for such partitions, save for the case of consecutive integer partitions. We now show that at least in the case of partitions with parts in arithmetic progression of common difference two, an enumeration formula, denoted $p_2(n)$, exists and is given in terms of the number of positive divisors $d(n)$ of n.

Lemma 1. *For any integer $n > 1$, the number of partitions of n with parts in arithmetic progression having a common difference of two is given by*

$$p_2(n) = \frac{1}{2}\left(d(n) - 2 + \frac{(-1)^{d(n)+1} + 1}{2}\right). \tag{3}$$

Proof. Recall $a + (a+2) + \cdots + (a + 2(n-1)) = n(n+a-1)$ and for the partitions in question $a, n \in \mathbb{N}$ with $a \geq 1$ and $n \geq 2$. Consequently, for an integer $N \geq 2$ the value of $p_2(N)$ is equal to the number of representations of the form $N = n(n+k)$, where $n \geq 2$ and $k \geq 0$. Now as $n \geq 2$ and $n + k \geq n$ our task is thus reduced to determining the number of divisors s of N such that $s \neq 1, N$ and $s \leq N/s$. If N is not a perfect square then $d(N)$ is even. Excluding the divisors $s = 1, N$ we see, after grouping the remaining $d(N) - 2$ divisors into pairs of the form $(s, N/s)$, that there must be exactly $(d(N) - 2)/2$ divisors satisfying the above condition. If

$N > 1$ is square then $d(N)$ is odd and $s = \sqrt{N} = N/s$. Thus after excluding the divisors $s \in \{1, \sqrt{N}, N\}$ and again pairing, there must be $(d(N) - 3)/2$ divisors s such that $s < N/s$. However, including again $s = \sqrt{N}$, one deduces that there are $(d(N) - 1)/2$ divisors $s \le N/s$ such that $s \ne 1, N$. Hence in both cases $p_2(N) = \frac{1}{2}(d(N) - 2 + ((-1)^{d(N)+1} + 1)/2)$.

Remark 1. *If the generating function of $p_2(n)$ is denoted by $f(q)$, that is $f(q) := \sum_{n=2}^{\infty} p_2(n)q^n$, observe that as the coefficient of q^N in the power series expansion of $q^{n^2}/(1 - q^n)$ is equal to the number of representations of $N = n(n + k)$, where $k \ge 0$ and $n \ge 2$, then*

$$f(q) = \sum_{n=2}^{\infty} \frac{q^{n^2}}{1 - q^n}.$$

In what follows recall that if the 3-tuple of integers (x, y, z) is such that $0 < x < y < z$ and $x^2 + y^2 = z^2$ then, we do not count the 3-tuple (y, x, z) as a Pythagorean triple having either x or y as one of its base lengths. By applying Lemma 1 we can now derive the desired enumeration formula.

Theorem 1. *For any integer $n > 1$ the number of Pythagorean triples in which n appears as one of the base lengths is given by*

$$\tfrac{1}{2}(d_0(n^2) + (-1)^{n+1} d_1(n^2) - 1). \tag{4}$$

Proof. In order to establish (4) our first aim will be to develop an enumeration formula, denoted $s(n)$, which counts the number of representations of an integer $n > 1$ as a difference of squares of two non-negative integers. Once this is achieved we shall deduce (4) from $s(n^2) - 1$, since each representation of the form $n^2 = x^2 - y^2$, having $x, y > 0$, yields a unique Pythagorean triple, where n is one of the base lengths, with the exception of the degenerate case $n^2 - 0^2$.

To begin, we make the following simple observation that the $p_2(n)$ partitions of n must have parts that are either all odd or all even consecutive integers. Denoting the number of partitions having entirely even or odd parts by $\phi(n)$ and $\sigma(n)$ respectively one has $p_2(n) = \phi(n) + \sigma(n)$. Now for $n > 2$ and even, there are exactly $d_1(n/2) - 1 = d_1(n) - 1$ consecutive integer partitions of $\frac{n}{2}$, of the form $\sum_{r=m}^{p} r$ with $p > m$. Consequently, there must be $d_1(n) - 1$ partitions of n of the form $\sum_{r=m}^{p} 2r$ and so $\phi(n) = d_1(n) - 1$. Of course, when n is odd, $\phi(n) = 0$ and so one can set $\phi(n) = (((-1)^n + 1)/2)(d_1(n) - 1)$. Thus from the above decomposition of $p_2(n)$ together with Lemma 1 we find that

$$\begin{aligned}\sigma(n) &= \frac{1}{2}\left(d(n) - 2 + \frac{(-1)^{d(n)+1} + 1}{2}\right) - \frac{(-1)^n + 1}{2}(d_1(n) - 1) \\ &= \frac{1}{2}\left(d_0(n) + (-1)^{n+1} d_1(n) + \frac{(-1)^{d(n)+1} + 1}{2}\right) + \frac{(-1)^n - 1}{2},\end{aligned}$$

where we have used the fact that $d(n) = d_0(n) + d_1(n)$. Recalling that the nth perfect square is equal to the sum of the first n consecutive odd integers, it is clear that each $\sigma(n)$ partition of n must correspond to a unique representation of the form $x^2 - y^2$, where $x, y \in \mathbb{N}$ (the set of non-negative integers). Since by definition each of the $\sigma(n)$ partitions contains at least two summands, we must have $x - y > 1$. However, when $n = 2r + 1$, for some $r \in \mathbb{N}$, one of the $s(n)$ representations must be of the form $n = (r+1)^2 - r^2$, thus $s(n) = \sigma(n) + 1$. If $n = 2r$ then clearly no such difference of consecutive squares representation can exist and so $s(n) = \sigma(n)$. Thus one may set $s(n) = \sigma(n) + (((-1)^{n+1} + 1)/2)$, which for $n > 2$ yields

$$s(n) = \frac{1}{2}\left(d_0(n) + (-1)^{n+1} d_1(n) + \frac{(-1)^{d(n)+1} + 1}{2}\right). \tag{5}$$

However, we also see that (5) holds for $n = 2$ since clearly 2 cannot be expressed as a difference of two squares and formally setting $n = 2$ in (5) gives $s(2) = 0$. Finally, recalling that $d(n^2)$ is always odd, observe from (5) that $s(n^2) - 1$ reduces to the expression in (4), as $n^2 + 1$ and $n + 1$ are of the same parity.

To illustrate Theorem 1 consider firstly the odd integer $n = 21 = 3 \cdot 7$, from which we readily see that $d_0(n^2) = 0$ while $d_1(n^2) = 9$. Thus from (4) there must be exactly four Pythagorean triples having a base length of 21. These are $(21, 28, 35)$, $(20, 21, 29)$, $(21, 72, 75)$ and $(21, 220, 221)$. Secondly, considering the even integer $n = 30 = 2 \cdot 3 \cdot 5$, we readily again see that $d_0(n^2) = 18$ while $d_1(n^2) = 9$. Thus from (4) there must also be exactly four Pythagorean triples having a base length of 30. These are $(30, 72, 78)$, $(16, 30, 34)$, $(30, 40, 50)$ and $(30, 224, 226)$.

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On the evaluation of determinants using two order subdeterminants

Edvard Kramar*

Abstract

Some simple algorithms are given for evaluation of determinants using only the second-order subdeterminants together with some illustrative examples.

The traditional methods for hand-calculation of the determinant of an $n \times n$ matrix are based either on simplifying the matrix by performing elementary row or column operations or on expansion by minors along some row or column. A brief overview of the theory of determinants can be found, for example, in [6] and [7]. There are many commonly-used computer packages such as Mathematica or Matlab in which the algorithms to find the determinant of a matrix are based on factorisation in a product of lower and upper matrices. The purpose of this note is to present some methods for evaluation of determinants using only second-order subdeterminants.

Some methods of evaluation of determinant

For a given $n \times n$ matrix, A, fix one of its nonzero element a_{ik} and denote it by α:

$$A = \begin{bmatrix} a_{11} & \dots & a_{1,k-1} & a_{1k} & a_{1,k+1} & \dots & a_{1n} \\ a_{21} & \dots & a_{2k-1} & a_{2k} & a_{2,k+1} & \dots & a_{2n} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & \dots & a_{i,k-1} & \alpha & a_{i,k+1} & \dots & a_{in} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{n,k-1} & a_{nk} & a_{n,k+1} & \dots & a_{nn} \end{bmatrix}. \tag{1}$$

Proposition 1. *For the determinant of the matrix A the following relation holds*

$$\det(A) = \frac{1}{\alpha^{n-2}} \det(C), \tag{2}$$

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where C is a matrix of the following form:

$$
\begin{bmatrix}
\begin{vmatrix} a_{11} & a_{1k} \\ a_{i1} & \alpha \end{vmatrix} & \dots & \begin{vmatrix} a_{1,k-1} & a_{1k} \\ a_{i,k-1} & \alpha \end{vmatrix} & \begin{vmatrix} a_{1k} & a_{1,k+1} \\ \alpha & a_{i,k+1} \end{vmatrix} & \dots & \begin{vmatrix} a_{1k} & a_{1n} \\ \alpha & a_{in} \end{vmatrix} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\begin{vmatrix} a_{i-1,1} & a_{i-1,k} \\ a_{i1} & \alpha \end{vmatrix} & \dots & \begin{vmatrix} a_{i-1,k-1} & a_{i-1,k} \\ a_{i,k-1} & \alpha \end{vmatrix} & \begin{vmatrix} a_{i-1,k} & a_{i-1,k+1} \\ \alpha & a_{i,k+1} \end{vmatrix} & \dots & \begin{vmatrix} a_{i-1,k} & a_{i-1,n} \\ \alpha & a_{in} \end{vmatrix} \\
\begin{vmatrix} a_{i1} & \alpha \\ a_{i+1,1} & a_{i+1,k} \end{vmatrix} & \dots & \begin{vmatrix} a_{i,k-1} & \alpha \\ a_{i+1,k-1} & a_{i+1,k} \end{vmatrix} & \begin{vmatrix} \alpha & a_{i,k+1} \\ a_{i+1,k} & a_{i+1,k+1} \end{vmatrix} & \dots & \begin{vmatrix} \alpha & a_{in} \\ a_{i+1,k} & a_{i+1,n} \end{vmatrix} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\begin{vmatrix} a_{i1} & \alpha \\ a_{n1} & a_{nk} \end{vmatrix} & \dots & \begin{vmatrix} a_{i,k-1} & \alpha \\ a_{n,k-1} & a_{nk} \end{vmatrix} & \begin{vmatrix} \alpha & a_{i,k+1} \\ a_{nk} & a_{n,k+1} \end{vmatrix} & \dots & \begin{vmatrix} \alpha & a_{in} \\ a_{nk} & a_{nn} \end{vmatrix}
\end{bmatrix}.
$$

Proof. Let us perform on columns c_j, $j \neq k$, of the matrix A the following replacements: $c_j \to \alpha.c_j - a_{ij}.c_k$. Then we obtain the following matrix:

$$
\begin{bmatrix}
\alpha a_{11} - a_{1k}a_{i1} & \dots & \alpha a_{1,k-1} - a_{1k}a_{i,k-1} & a_{1k} & \alpha a_{1,k+1} - a_{1k}a_{i,k+1} & \dots & \alpha a_{1n} - a_{1k}a_{in} \\
\alpha a_{21} - a_{2k}a_{i1} & \dots & \alpha a_{2,k-1} - a_{2k}a_{i,k-1} & a_{2k} & \alpha a_{2,k+1} - a_{2k}a_{i,k+1} & \dots & \alpha a_{2n} - a_{2k}a_{in} \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \dots & 0 & \alpha & \dots & \dots & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha a_{n1} - a_{nk}a_{i1} & \dots & \alpha a_{n,k-1} - a_{nk}a_{i,k-1} & a_{nk} & \alpha a_{n,k+1} - a_{nk}a_{i,k+1} & \dots & \alpha a_{nn} - a_{nk}a_{in}
\end{bmatrix}.
$$

The kth column is unchanged, in the ith row all elements are 0 except on the kth position. The determinant of the matrix A is then equal to the determinant of this matrix divided by α^{n-1}. Let us expand the determinant of the last matrix by the ith row. We take into account the factor $(-1)^{i+k}$ by multiplying the last $n-i$ rows and the last $n-k$ columns by -1. The determinant of the matrix A is then $1/(\alpha^{n-2})$ times the determinant of the following matrix:

$$
\begin{bmatrix}
\alpha a_{11} - a_{1k}a_{i1} & \dots & \alpha a_{1,k-1} - a_{1k}a_{i,k-1} & a_{1k}a_{i,k+1} - \alpha a_{1,k+1} & \dots & a_{1k}a_{in} - \alpha a_{1n} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\alpha a_{i-1,1} - a_{i-1,k}a_{i1} & \dots & \alpha a_{i-1,k-1} - a_{i-1,k}a_{i,k-1} & a_{i-1,k}a_{i,k+1} - \alpha a_{i-1,k+1} & \dots & a_{i-1,k}a_{in} - \alpha a_{i-1,n} \\
a_{i+1,k}a_{i1} - \alpha a_{i+1,1} & \dots & a_{i+1,k}a_{i,k-1} - \alpha a_{i+1,k-1} & \alpha a_{i+1,k+1} - a_{i+1,k}a_{i,k+1} & \dots & \alpha a_{i+1,n} - a_{i+1,k}a_{in} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
a_{nk}a_{i1} - \alpha a_{n1} & \dots & a_{nk}a_{i,k-1} - \alpha a_{n,k-1} & \alpha a_{n,k+1} - a_{nk}a_{i,k+1} & \dots & \alpha a_{nn} - a_{nk}a_{in}
\end{bmatrix}
$$

and this is in fact the matrix C. The proof is complete.

Thus, the determinant of the matrix A of the nth order can be reduced to the evaluation of the determinant of a matrix of order $n-1$. Each element of this reduced matrix is the second-order subdeterminant of the elements in A lying on the intersections of row i and column k with a particular row and column, which always includes the chosen element α. The positions remain the same as in the original matrix without taking care of the signs.By successively applying this formula we come to only one second-order determinant. By hand-calculation, it is practical to choose the element 1 or -1, if any.

As an example, let us evaluate the following determinant from [1]:

$$\begin{vmatrix} 2 & -1 & 5 & 8 & 3 & -4 \\ 0 & 4 & -3 & 4 & 3 & 8 \\ 1 & -3 & -2 & 5 & 7 & \mathbf{1} \\ 4 & 6 & -4 & 2 & 9 & 0 \\ 3 & 5 & -2 & 4 & 7 & 0 \\ 2 & 4 & 6 & -3 & 2 & 8 \end{vmatrix} = \begin{vmatrix} 6 & -13 & -3 & 28 & 31 \\ -8 & 28 & 13 & -36 & -53 \\ -4 & -6 & 4 & -2 & -9 \\ -3 & -5 & \mathbf{2} & -4 & -7 \\ 6 & -28 & -22 & 43 & 54 \end{vmatrix}$$

$$= \frac{1}{2^3}\begin{vmatrix} 3 & -41 & -44 & -41 \\ 23 & 121 & 20 & 15 \\ \mathbf{4} & 8 & -12 & -10 \\ 54 & 166 & -2 & -46 \end{vmatrix}$$

$$= \frac{1}{2^3 4^2}\begin{vmatrix} 188 & 140 & 134 \\ \mathbf{-300} & -356 & -290 \\ 232 & 640 & 356 \end{vmatrix}$$

$$= \frac{-1}{128 \times 300}\begin{vmatrix} -24928 & -14320 \\ -109408 & -39520 \end{vmatrix}$$

$$= 15145.$$

We denoted in bold the elements used in the next step. In hand-calculation it is always practical to reduce the fractions by common factors in some row or column. Note that if we choose in the above relation $\alpha = a_{11}$ we obtain Chio's formula (see [5]). For the evaluation of the determinant of a nth order matrix, the required multiplication and division operations are $(4n^3 - 3n^2 - 7n + 6)/6$ which is a little less than for the algorithm in [1]. If we add the subtraction operations, the total sum of arithmetic operation to be used is $n^3 - n^2 - n + 1$.

The above formula can be generalised by choosing two or more nonzero elements in some row. For example, let us choose $\alpha = a_{ik} \neq 0$ and $\beta = a_{is} \neq 0$, where $1 \leq k < s \leq n$. Similar to above, we obtain the new formula by replacing the columns of the original matrix A in the following manner:

$$\begin{aligned} c_j &\to \alpha \cdot c_j - a_{ij} \cdot c_k \qquad && j = 1, 2, \ldots, s,\ j \neq k, \\ c_j &\to \beta \cdot c_j - a_{ij} \cdot c_s \qquad && j = s+1, \ldots, n. \end{aligned}$$

Proposition 2. *For a given matrix A in (1) let us choose $\alpha = a_{ik} \neq 0$ and $\beta = a_{is} \neq 0$. Then the following formula holds:*

$$\det(A) = \frac{1}{\alpha^{s-2}\beta^{n-s}} \det(C), \tag{3}$$

where C is the matrix consisting of second-order subdeterminants with a similar form to that in Proposition 1, where the first parameter α is used to form the first $s-1$ columns, while for the remaining columns we use the parameter β.

As an example, let A be a 5×5 matrix with the two chosen nonzero elements $\alpha = a_{32}$ and $\beta = a_{34}$:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & \alpha & a_{33} & \beta & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

Then the following formula holds:

$$\det(A) = \frac{1}{\alpha^{4-2}\beta^{5-4}} \begin{vmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & \alpha \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ \alpha & a_{33} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{14} \\ \alpha & \beta \end{vmatrix} & \begin{vmatrix} a_{14} & a_{15} \\ \beta & a_{35} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & \alpha \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} \\ \alpha & a_{33} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{24} \\ \alpha & \beta \end{vmatrix} & \begin{vmatrix} a_{24} & a_{25} \\ \beta & a_{35} \end{vmatrix} \\ \begin{vmatrix} a_{31} & \alpha \\ a_{41} & a_{42} \end{vmatrix} & \begin{vmatrix} \alpha & a_{33} \\ a_{42} & a_{43} \end{vmatrix} & \begin{vmatrix} \alpha & \beta \\ a_{42} & a_{44} \end{vmatrix} & \begin{vmatrix} \beta & a_{35} \\ a_{44} & a_{45} \end{vmatrix} \\ \begin{vmatrix} a_{31} & \alpha \\ a_{51} & a_{52} \end{vmatrix} & \begin{vmatrix} \alpha & a_{33} \\ a_{52} & a_{53} \end{vmatrix} & \begin{vmatrix} \alpha & \beta \\ a_{52} & a_{54} \end{vmatrix} & \begin{vmatrix} \beta & a_{35} \\ a_{54} & a_{55} \end{vmatrix} \end{vmatrix}.$$

The above relations can be generalised to

Proposition 3. *Let us choose in the ith row of the matrix (1) the nonzero parameters $a_{i,k_1}, a_{i,k_2}, \dots, a_{i,k_r}$, where $1 \le k_1 < k_2 < \cdots < k_r < n$, then the following formula holds:*

$$\det(A) = \frac{1}{a_{i,k_1}^{k_2-2} a_{i,k_2}^{k_3-k_2} \dots a_{i,k_r}^{n-k_r}} \det(C), \tag{4}$$

where C is a matrix of 2×2 determinants formed, as in the previous propositions, progressively using the chosen parameters a_{i,k_j}, skipping to the next parameter at the k_{j+1}th column.

In particular, we can choose all elements of the first row, apart from the first and last: a_{ij}, $j = 2, 3\dots, n-1$, if all these elements are nonzero. In this case we obtain the following formula:

$$\det(A) = \frac{1}{a_{i2}a_{i3}\dots a_{i,n-1}} \det(C), \tag{5}$$

where C is the matrix of 2×2 determinants formed in the above way, skipping to the next chosen parameter with each step.

It is interesting that the position of the first parameter in (3) and (4) has no impact on the above factor. The similar formulas as (3), (4) and (5) hold for the parameters chosen in some column.

As an example let us use the relation (5) to evaluate the following determinant of order n

$$D(a_1,a_2,\dots,a_n;b_1,b_2,\dots,b_n)=\begin{vmatrix} a_1^{n-1} & a_1^{n-2}b_1 & \dots & a_1b_1^{n-2} & b_1^{n-1}\\ a_2^{n-1} & a_2^{n-2}b_2 & \dots & a_2b_2^{n-2} & b_2^{n-1}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ a_n^{n-1} & a_n^{n-2}b_n & \dots & a_nb_n^{n-2} & b_n^{n-1}\end{vmatrix} \tag{6}$$

Firstly, we assume that all parameters are nonzero. Using the identity (5) for the first row, we obtain a determinant of order $n-1$. Reducing all common factors in powers of a_1 and b_1 from all columns of this determinant and factoring out all common factors from each row we obtain

$$D(a_1,a_2,\dots,a_n;\, b_1,b_2,\dots,b_n)=(a_1b_2-a_2b_1)(a_1b_3-a_3b_1)\dots(a_1b_n-a_nb_1) \times D(a_2,a_3,\dots,a_n;b_2,b_3,\dots,b_n). \tag{7}$$

Continuing this way we have to evaluate only one second-order determinant

$$D(a_{n-1},a_n;\, b_{n-1},b_n)=\begin{vmatrix} a_{n-1} & b_{n-1}\\ a_n & b_n\end{vmatrix}=(a_{n-1}b_n-a_nb_{n-1}).$$

Thus, we obtain the identity

$$D(a_1,a_2,\dots,a_n;\, b_1,b_2,\dots,b_n)=\prod_{1\le i<j\le n}(a_ib_j-a_jb_i). \tag{8}$$

This relation also holds in the case that some of the numbers are equal to zero. We may show this for the case $a_1=0$. Expanding the determinant (6) by the first row we obtain in this case

$$D(a_1,a_2,\dots,a_n;\, b_1,b_2,\dots,b_n)=(-1)^{n+1}b_1^{n-1}a_2a_3\dots a_n \times D(a_2,a_3,\dots,a_n;\, b_2,b_3,\dots,b_n).$$

The same relation follows from (7) taking $a_1=0$. Note, that the above determinant can be evaluated using the formula for the Vandermonde determinant (see [5]). On the other hand the identity for the Vandermonde determinant follows from (8) taking $b_i=1$ for all i.

Application to a Hessenberg matrix

The determinants of some Hessenberg matrices were treated by several authors (see e.g. [2], [3], [8]). We shall use relation (2) to compute the determinant of a Hessenberg matrix of the form:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1,n-1} & a_{1n}\\ a_{21} & a_{22} & a_{23} & \dots & a_{2,n-1} & a_{2n}\\ 0 & a_{32} & a_{33} & \dots & a_{3,n-1} & a_{3n}\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \dots & a_{n-1,n-1} & a_{n-1,n}\\ 0 & 0 & 0 & \dots & a_{n,n-1} & a_{nn}\end{bmatrix}.$$

Let $a_{11}\neq 0$, then it is easy to see, using one step of the algorithm in Proposition 1, that we have in fact only in the first row the second-order subdeterminants, all remaining rows are only multiplied by factor a_{11}. If we factor this out, we obtain

$$\det(A) = \det \begin{bmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & \cdots & \begin{vmatrix} a_{11} & a_{1,n-1} \\ a_{21} & a_{2,n-1} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{1n} \\ a_{21} & a_{2n} \end{vmatrix} \\ a_{32} & a_{33} & \cdots & a_{3,n-1} & a_{3n} \\ 0 & a_{43} & \cdots & a_{4,n-1} & a_{4n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & a_{n-1,n-1} & a_{n-1,n} \\ 0 & 0 & \cdots & a_{n,n-1} & a_{nn} \end{bmatrix}. \quad (9)$$

If $a_{11} = 0$ and $a_{21} \neq 0$ we start with this element and we obtain the same relation. If both the terms are equal to 0, the above relation holds trivially. We see that the application of the above relation is very simple. In each step we evaluate only the second-order subdeterminants from the first two rows, the other rows remains the same, and we omit the 0s in the first column. The new matrix is again of the Hessenberg form and we continue this way to come to only one determinant of the second order. A similar relation can be obtained starting from the last lower element.

For example:

$$\begin{vmatrix} 1 & 1 & 3 & 2 & 1 \\ 1 & 2 & 4 & 5 & 2 \\ 0 & 3 & 5 & 6 & 5 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 9 & 8 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 3 & 1 \\ 3 & 5 & 6 & 5 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 9 & 8 \end{vmatrix} = \begin{vmatrix} 2 & -3 & 2 \\ 1 & 2 & 4 \\ 0 & 9 & 8 \end{vmatrix} = \begin{vmatrix} 7 & 6 \\ 9 & 8 \end{vmatrix} = 2.$$

In a further example, let $A_{n,x}$ be the $n \times n$ Hessenberg matrix, where $n \geq 3$, in which the elements on the main diagonal are 2s, the elements on the sub- and superdiagonal are 1s, the element in $(1, n)$ is x, for some $x \in \mathbb{C}$, and all others element are 0. For $n = 10$, the determinant position of such a matrix is evaluated in [8] by some algorithm for evaluation of tridiagonal determinats (see also [4]). Let B_n be a tridiagonal matrix obtained if $x = 0$ with the $(1, 2)$ element in the first matrix replaced by 2. For $n = 5$ this matrices are

$$A_{5,x} = \begin{bmatrix} 2 & 1 & 0 & 0 & x \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}, \qquad B_5 = \begin{bmatrix} 2 & 2 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

For $a_{n,x} := \det(A_{n,x})$ and $b_n := \det(B_n)$ it is easy to see that:

$$b_n = 2 \quad n \geq 2, \qquad a_{n,x} = n + 1 + (-1)^{n-1}x \quad n \geq 3.$$

Namely, if we apply (9) to b_n we obtain the same form of the order $n - 1$, thus $b_n = b_{n-1}$, and since $b_2 = 2$, we are done for b_n. If we apply (9) to $a_{n,x}$ twice and expand the first row in two summands, we obtain the relation $a_{n,x} = a_{n-2,x} + b_{n-2} = a_{n-2,x} + 2$. Hence for the induction step we have $a_{n+1,x} = a_{n-1,x} + 2 = n + (-1)^{n-2}x + 2 = n + 2 + (-1)^n x$. Since $a_{3,x} = 4 + x$ and $a_{4,x} = 5 - x$, this formula holds for all $n \geq 3$.

In [2] and [3] some Hessenberg matrices are studied with determinants that are Fibonacci numbers. As an example let $H_{n,t}$ be the $n \times n$ Hessenberg matrix in which subdiagonal entries are -1s, the main diagonal entries, except the last one, are 2s, and the entries of each column above the main diagonal alternate between -1s and 1s, starting with -1. The lowest diagonal element is equal to $t+1$, for some $t \in \mathbb{C}$. In [3] the determinant of this matrix was shown to be $h_{n,t} := \det(H_{n,t}) = f_n + t f_{n+1}$, where f_n are Fibonacci numbers. The proof was obtained using a system of two recursive relations for the above determinant and for the determinant of a similar matrix where the $(1,1)$ entry is replaced with 1. In [2] a combinatorial proof was given. If we use the relation (9) to the matrix $H_{n,t}$ and expand the first column in two summands, we obtain directly the relation $h_{n,t} = h_{n-1,t} + h_{n-2,t}$ and then using the properties of Fibonacci numbers we obtain the result by induction.

A special case of a Hessenberg matrix is the tridiagonal matrix

$$A = \begin{bmatrix} b_1 & c_1 & 0 & \dots & 0 & 0 \\ a_2 & b_2 & c_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & b_{n-1} & c_{n-1} \\ 0 & 0 & 0 & \dots & a_n & b_n \end{bmatrix}.$$

To find its determinant we can use the above algorithm where in each step we calculate two new elements. This can be organised also as the following recursive calculations:

$$\begin{aligned} d_1 &= b_1, & e_1 &= c_1, & & \\ d_r &= b_r d_{r-1} - a_r e_{r-1}, & e_r &= c_r d_{r-1}, & r &= 2, 3, \dots, n-1, \\ d_n &= b_n d_{n-1} - a_n e_{n-1}. & & & & \end{aligned}$$

The last value is the determinant of our matrix: $\det(A) = d_n$. This recursion can also be written as: $d_r = b_r d_{r-1} - a_r c_{r-1} d_{r-2}$, $r = 3, 4, \dots, n$, where $d_1 = b_1$, $d_2 = b_2 d_1 - a_2 c_1$. For example, if we apply this recursion to the above matrix B_n we obtain by induction again $\det(B_n) = 2$, for all n.

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Book reviews

The Computer as Crucible:
An Introduction to Experimental Mathematics

Jonathan Borwein and Keith Devlin
A.K. Peters, 2008, ISBN-13: 978-1568813431

Whenever a book's preface states its aims, a natural question to ask is whether it succeeds in meeting them. Keith Devlin and Jonathan Borwein, two mathematicians with expertise in different mathematical fields but with a common interest in experimental mathematics, begin this book by saying:

> Our aim in writing this book was to provide a short, readable account of experimental mathematics. It is not intended as a textbook to accompany a course ... In particular, we do not aim for comprehensive coverage of the field; rather, we pick and choose topics and examples to give the reader a good sense of the current state of play in the rapidly growing field of experimental mathematics.

The sleuth-like style and lucid writing certainly make this book an enjoyable read. Many explanations are framed by relevant historical context and tales of mathematicians whose use of experimental mathematics helped them gain insights into difficult problems. Although it was never intended to be a course textbook, it could be used as a supplementary text. Many of the chapters are short, and should be viewed as aperitifs.

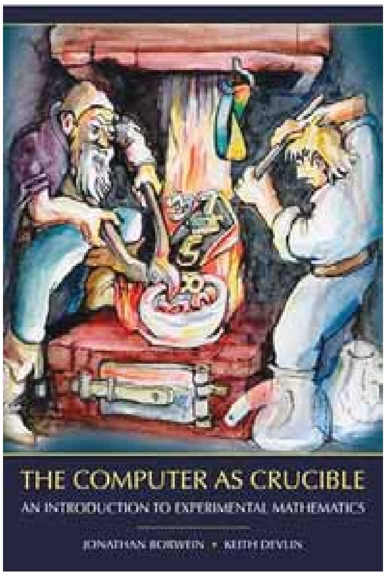
Chapter 1 deals with the important question 'What is experimental mathematics?'. In the authors' own words:

> Experimental mathematics is the use of a computer to run computations — sometimes no more than trial-and-error tests — to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions that may themselves arise by computational means, including search.

Broadly speaking, it is the use of computers in mathematics as tools in their own right, not simply as numerical calculation aids, '... experimentation is regarded as a significant part of mathematics in its own right ...'.

What kind of experimentation? Here are some of the things described in this book:

- symbolic computation using a computer algebra system such as Maple or Mathematica
- data visualisation
- integer-relation algorithms like PSLQ
- high-precision integer and floating-point arithmetic
- high-precision evaluation of integrals and summation of infinite series
- identification of functions based on their graph characteristics.



It would be very easy to fall into the belief that great mathematicians pluck profound and deep results out of thin air, but some of the mathematical greats (Gauss, Euler, Fermet, Riemann, ...) were confirmed experimenters who would spend many hours carrying out calculations in order to discover new mathematical avenues worth pursuing. The 72-year-old Gauss recounted in a letter to the astronomer, Johann Encke, that, as a young boy of 15, armed with a table of logarithms he 'frequently spent an idle quarter of an hour to count another chiliad here and there' [1], which led to his estimate of the density of prime numbers; "... Gauss was very clearly an 'experimental mathematician' of the first order."

Chapter 2 gives a brief introduction to the PSLQ algorithm, an integer relation algorithm developed by Helaman Ferguson. Given any real coefficients $\mathrm{a}_0, \mathrm{a}_1, \ldots, \mathrm{a}_n$ and a precision ε, an integer relation algorithm uses high-precision arithmetic to find integer coefficients $\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_n$ such that $\lambda_0 \neq 0$ and

$$|\lambda_0 \mathrm{a}_0 + \lambda_1 \mathrm{a}_1 + \cdots + \lambda_n \mathrm{a}_n| < \varepsilon$$

or else it tells you such expression exists within a ball of a given radius about the origin.

Chapter 3 ('What Is That Number?') introduces inverse symbolic calculators as tools to recognise numbers, and combined with Sloane's online *Encyclopedia of Integer Sequences*, describes a technique for determining closed forms of sequences.

I have more than a passing interest in Riemann's zeta function, the topic of Chapter 4 ('The Most Important Function in Mathematics'); I found it interesting though perhaps a little short. I particularly liked the quote about British soccer player, Wayne Rooney, contrasting him with David Beckham: 'There is more chance of him [Rooney] proving Riemann's Hypothesis than wearing a sarong'!

I'm certain physicists will find Chapter 5 ('Evaluate the Following Integral') interesting, especially given the authors' collaborations in computing closed forms of definite integrals arising in physics.

Chapter 9 ('Take It to the Limit') contains three worked examples of finding closed forms for infinite sums, and Chapter 10 ('Danger! Always Exercise Caution When Using the Computer') contains sobering stories and examples of some of the pitfalls faced by experimental mathematicians. Here is one:

$$\begin{aligned}\operatorname{sinc}(x) &= \frac{\sin x}{x},\\ I_1 &= \int_0^\infty \operatorname{sinc}(x)\,\mathrm{d}x,\\ I_2 &= \int_0^\infty \operatorname{sinc}\left(\tfrac{x}{3}\right)\mathrm{d}x,\\ I_3 &= \int_0^\infty \operatorname{sinc}(x)\operatorname{sinc}\left(\tfrac{x}{3}\right)\operatorname{sinc}\left(\tfrac{x}{5}\right)\mathrm{d}x, \quad \ldots\\ I_8 &= \int_0^\infty \operatorname{sinc}(x)\operatorname{sinc}\left(\tfrac{x}{3}\right)\operatorname{sinc}\left(\tfrac{x}{5}\right)\ldots\operatorname{sinc}\left(\tfrac{x}{15}\right)\mathrm{d}x.\end{aligned}$$

A computer algebra system (CAS) will discover that $I_1 = I_2 = I_3 = \cdots = I_7 = \pi/2$ but $I_8 = 0.499999999992646\pi$.

On finding this, the authors suspected a bug in the CAS software. But there is no bug!

The book provides tantalising examples and suggestions to whet the reader's appetite in the form of an 'Explorations' section at the end of each chapter. These are not exactly exercises but there is a corresponding 'Answers and Reflections' chapter at the end of the book. Interested readers will find many of these topics expanded upon in [2].

I thoroughly enjoyed reading this short introduction to experimental mathematics. It will no doubt appeal to a broad mathematical audience, both professional and amateur alike. If I have one complaint (well, more of a request), it would be a much longer chapter on evaluating definite integrals! But then, in the words of G.H. Hardy, 'I could never resist a definite integral' [3].

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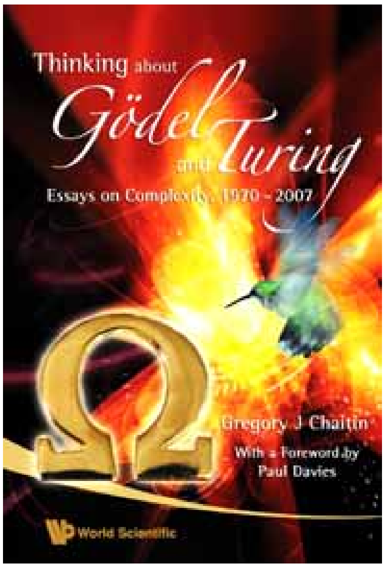
Thinking About Gödel and Turing: Essays on Complexity, 1970–2007

Gregory J. Chaitin
World Scientific Publishing, 2007, ISBN-13 978-981-270-895-3

Gregory Chaitin's *Thinking About Goedel and Turing: Essays on Complexity, 1970–2007* contains 23 papers, all by Chaitin, on a wide range of topics. Although the theme of complexity is common to all the papers, many other related issues are also discussed. Chaitin has things to say about Gödel's theorem, about randomness in arithmetic, the foundations of mathematics, the role of complexity and simplicity in scientific method and their connection to the notion of explanation, the similarities between mathematics and physics and his own 'digital philosophy'.

Perhaps Chaitin's best-known contribution to mathematics is his demonstration of the existence of the so-called 'Ω' number, which gives the probability that any randomly chosen computer programme will eventually halt. Chaitin has shown that the binary expansion of Ω is random in the sense that there can be no rule for generating this sequence of '1's and '0's that is shorter than the sequence itself.



Chaitin goes on to draw a number of broadly 'philosophical' conclusions from this. In particular, Chaitin feels that mathematicians have not taken Gödel's Incompleteness Theorems sufficiently seriously. He says that mathematicians tend to 'carry on' with their own mathematical work as before. But for Chaitin, this is a mistake. He argues that 'incompleteness ... [in mathematics] is natural and pervasive'. He sees certain aspects of the history of mathematics as supporting this idea. In particular, he points out (p. 72) that 'many mathematical problems have remained un-solved for hundreds and even thousands of years' and that the reason for this might not lie within mathematicians themselves, but might rather be due to 'the incompleteness of their axioms'. He goes on to suggest that mathematicians should therefore adopt a more liberal attitude to what is to count as an axiom. (He suggests 'There are no odd perfect numbers' as a possible candidate. He also mentions the Riemann hypothesis.)

These ideas lead Chaitin to develop a view of mathematics according to which the subject resembles empirical sciences more closely than is generally thought. In the empirical sciences, laws are not accepted because they are 'self-evident' or '(epistemically) necessary' or 'analytically true'; rather, they are accepted because they explain a lot of other things. Roughly, they are accepted because a lot of other statements we know to be true follow from them. Chaitain suggests that mathematicians ought to view axioms in a way similar to the way scientists view laws: they are to be accepted if we know they have a lot of true consequences. Chaitin acknowledges, however, that most mathematicians would see this suggestion as 'ridiculous'.

Another recurring theme in the book is simplicity and complexity in science. Chaitin notes that since the Ancient Greeks, scientists have believed nature is simple. He produces a number of very interesting quotes, from Plato through to more recent figures such as Weyl, Einstein and Feynman who all agree that it is very important that our theories of the physical world be as simple as possible. He pays special attention to Leibniz, and acknowledges him as a precusor of some of his own ideas.

Chaitin asks: 'What, precisely, is simplicity?' and 'Why is it so desirable?' Briefly, his answer to the first question is that simplicity is compressibility; his answer to the second is that compressibility gives comprehension. He writes: 'a scientific theory is a computer program, and the smaller, the more concise the program, the better the theory' (p. 211).

Chaitin notes that this view is based on a number of assumptions. One of the assumptions is that 'the choice of computer or programming language is not too important'. He goes on to note that 'This is debatable' (p. 212). However, I believe that there is a serious difficulty here that Chaitin has not sufficiently addressed. Empirical theories of the physical or material world will contain non-mathematical terms referring to classes or kinds of things. For example, the simple 'theory', 'All crows are black', contains the terms 'crow' and 'black'. Different languages will contain different terms. Are all *possible* languages to be permitted in expressing theories of the physical world? It is easy to see that if all possible languages are permitted, then simplicity becomes quite useless as a way discriminating between good theories and bad. Let us introduce a language that contains a term 'runcible'. It is possible to define 'runcible', but the definition is very complex. Let O, O*, O** and so on be all the material objects. (We do need to assume the number of material objects is countable.) Let P be the properties of O, P* be the properties of O* and so on. Then we can say X is runcible if and only if X is O and has P, or X is O* and has P*, and so on. Then 'All material objects are runcible' will tell us the truth about every material object. There is, moreover, a sense in which it is a very simple theory: it can be stated in five words of English. But it is a lousy theory.

Intuitively, what is wrong with 'All material objects are runcible' is that it is very complex, but the complexity is contained in the definition of runcible. We might therefore suggest that in evaluating the complexity of a theory, we must first translate it to genuine or legitimate words such as 'crow' or 'black'. But this raises a new question: "What is to count as a 'genuine' or 'legitimate' term?" This has proved enormously difficult to answer! This is not the place to review these difficulties, but some of the difficulties that have most exercised philosophers, at least, are discussed in Nelson Goodman's *Fact, Fiction and Forecast* (Harvard University Press, 1955, especially Chapter 3).

There are very many other ideas dealt with in Chaitin's book. The author's enthusiasm for mathematics, for ideas generally, and for life, comes through clearly. The writing style is informal, and much of it reads like transcripts of lectures and addresses. This book is perhaps not everyone's cup of tea, but for anyone who wants a book bubbling with ideas, written by an author with a passion for his subject, I would strongly recommend it.

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AMSI News

Philip Broadbridge*

What can mathematics gain from recent government initiatives?

In the Federal Budget papers for 2009–2010, there are no specific initiatives for mathematics. The word ‘mathematics’ appears only once[1], within a package of HECS HELP that has now been extended to all sciences, as well as nursing and teacher training:

> Consistent with the Government’s ongoing interest in supporting critical skills needs, the HECS HELP benefit that now applies to eligible mathematics, science and early childhood education graduates will be extended to nursing and education graduates who take up nursing and education occupations.

The Budget provides a welcome boost to science infrastructure, by $901 million. Mathematics, more than most other disciplines, should be well placed to receive spin-offs from significant boosts in infrastructure to other scientific disciplines such as marine science, meteorology, space science, biotechnology and nanotechnology. At the forefront of each of these disciplines, one finds interesting mathematical problems. Mathematical scientists, more than others, are required to adapt their skills to collaborate with other disciplines. Mathematics departments have no choice but to do this if they are to regain a presence anywhere near as strong as that of 30 years ago. That does not mean that the discipline should lose its own identity. The solution of difficult mathematical problems, even those that arise in applications, requires extensive specialist training as well as the self-discipline required in rigorous research.

In much less than a year, AMSI has brokered 12 agreements between universities, outside employers and postgraduate interns[2]. This is just the kind of collaborative activity encouraged by Terry Cutler, author of the government-commissioned 2009 report, *Venturous Australia: building strength in innovation*, a review of Australia’s innovation system. The AMSI Internship program has been partly sponsored by a grant from the Collaboration and Structural Reform Fund, now replaced by the Diversity and Structural Adjustment Fund of DEEWR.

Diversity, breadth of opportunity and regional access to higher education are strong themes in the 2009 report of the Bradley Review, *Transforming Australia’s Higher Education System*. From personal experience, shared by many of my colleagues, nothing empowers working-class kids more than a rigorous school mathematics education. With the stated aim of having 40% of young adults graduate with degrees, the schools need to lift their aims in mathematics education. The National Curriculum Board’s mathematics writing team of ten (including Michael Evans and

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[1] Australian Federal Budget Papers. 4. *Future Directions for Higher Education*
[2] See http://www.amsi.org.au/Industry_internships.php

Janine McIntosh of AMSI) and its larger directing team (including Peter Stacey as a university mathematician), has an opportunity to make a real improvement. Even if the new school curriculum succeeds, university teaching departments will need to re-tool to cater for a broadening range of student needs.

In previous columns, I have given many examples of AMSI's efforts to demonstrate the value of the mathematical sciences in many other fields of endeavour. However, unlike bricks, mortar, telescopes and fowl houses, mathematical expertise is not given due recognition as an important infrastructure for scientific and economic development. Mathematics has been a proud discipline in its own right over several thousand years of intellectual stimulus. Some bureaucrats of the 21st Century dare to question its status. Even as I write, a review by management in one of our member universities is proposing to destroy their only internationally recognised mathematics activity. In Russia, such cultural Philistines were likened by V.I. Arnold[3] as Krylov's fabled pigs under an oak tree, 'both eating the acorns and digging up its roots'. It would be easy to be dismissive of them except for the fact that they hold the purse strings.

Director of AMSI since 2005, Phil Broadbridge was previously a professor of applied mathematics for 14 years, including a total of eight years as department chair at University of Wollongong and at University of Delaware. His PhD was in mathematical physics (University of Adelaide). He has an unusually broad range of research interests, including mathematical physics, applied nonlinear partial differential equations, hydrology, heat and mass transport and population genetics. He has published two books and 100 refereed papers, including one with over 150 ISI citations. He is a member of the editorial boards of four journals and one book series.

[3]V.I. Arnold, *Innumeracy and the Fires of the Inquisition*, Izvestya, 17 January 1998.

News

General News

G.S. Watson Annual Lecture, 2009

'Do Australian parents want both a son and a daughter?', presented by Dr Rebecca Kippen (Australian Demographic and Social Research Institute, Australian National University), Thursday, 20 August 2009, 4.30pm to 5.30pm, Room 3.03, Applied Science 1 Building, Department of Mathematics and Statistics, La Trobe University Bendigo.

Research internationally has found that parents in many countries hold strong preferences when it comes to the sex of their children. In general, sons are preferred in India and China, while in Europe and English-speaking countries there is a bias towards at least one boy and one girl.

Using analysis of Australian survey and census data, Dr Kippen investigates whether Australian parents want 'one of each', whether women prefer daughters and men prefer sons (or vice versa), and what Australians think about sex-selection technology including the hypothetical 'blue' and 'pink' pills.

About the Presenter: Rebecca Kippen completed a Bachelor of Business at La Trobe University, Bendigo, before undertaking postgraduate study in Demography at the Australian National University. She is now a Fellow in the Australian Demographic and Social Research Institute at the Australian National University.

Rebecca's research interests include fertility modelling, Australian population futures and historical demography. Her work with Professor Peter McDonald and others has contributed significantly to the public debate in areas such as population growth and immigration. The research discussed in this lecture was funded by the Australian Research Council under grant DP0558818 and was carried out with Ann Evans and Edith Gray.

More information: www.latrobe.edu.au/maths/watson/

Australian Learning and Teaching Council Project

The ALTC (Australian Learning and Teaching Council, formerly the Carrick Institute) is funding the \$215 000 project 'A national discipline-specific professional development programme for lecturers and tutors in the mathematical sciences', submitted in the Leadership for Excellence category. This initiative is supported by the AustMS, will commence mid-2009 and be completed by mid-2011.

The project aims to develop a suitable program of induction and professional development, systematically embedding knowledge and learning and teaching skills for early- or mid-career academics and inspiring experienced academics to use new ideas. The project is lead by Dr Leigh Wood (Macquarie University), and supported by team members Professor Nalini Joshi (Sydney University), Dr Birgit Loch (University of Southern Queensland), Associate Professor Diane Donovan (University of Queensland), Professor Walter R. Bloom (Murdoch University), Dr Matt Bower (Macquarie University), Dr Natalie Brown (University of Tasmania) and Jane Skalicky (University of Tasmania).

Australian National University

Professor Vladimir Bazhanov will be a plenary speaker at the XVI International Congress of Mathematical Physics to be held in Prague, 3–8 August 2009, where he will give a talk on 'Quantum Geometry'. For further information, see the ICMP09 web site at http://www.icmp09.com.

Bulletin of the Australian Mathematical Society

Students and colleagues are reminded that the *Bulletin of the Australian Mathematical Society* publishes 'Abstracts of Australasian PhD Theses'.

The *Bulletin* will accept abstracts of up to three pages (which may include references), and the submission may be the abstract included in the thesis itself. All the mathematical sciences are covered, including pure and applied mathematics, probability and mathematical statistics, mathematical physics and mathematical computer science.

Complete details can be found on the Bulletin page of the Society's web site at http://www.austms.org.au/Bulletin.

Students who have recently completed their PhD are encouraged to consider publication in the Bulletin.

University of Ballarat

- Alex Kruger has taken over from Prabhu Manyem as local correspondent for the *Gazette*. The *Gazette* thanks Prabhu for his time as correspondent.
- Professor Jon Borwein, Visiting Conjoint Professor Laureate in Mathematics, University of Newcastle, is coming to Ballarat on 9 November to give the Alexander Rubinov Memorial Lecture for 2009: 'Computer as Crucible'.

University of New South Wales: Scholarships in Mathematics and Statistics 2009

Scholarships in Mathematics and Statistics for 2009 were awarded at the Faculty of Science Presentation Evening on the 22 April. Congratulations to all the recipients, who were:

- Trevor Rose, SAS Institute Scholarship;
- Oliver Nunn, H.C. & M.E. Porter Memorial Scholarship;

- Georgia Tsambos, Faculty Undergraduate Scholarship;
- Matthew Brassil, Bundilla Scholarship;
- Harley Scammell, Bundilla Scholarship;
- Timothy Yip, Alma Douglas Scholarship for Statistics;
- Dominic Catsaras, School of Mathematics & Statistics Award;
- Corey Lewis, School of Mathematics & Statistics Award;
- Jennifer Dang, Girls Do The Maths Scholarship;
- Daniel Floyd, School of Mathematics & Statistics Rural Scholarship;
- Matthew Perrett, School of Mathematics & Statistics Rural Scholarship;
- Sen Lin, School of Mathematics & Statistics Scholarship;
- Anthony Morris, School of Mathematics & Statistics Scholarship;
- Jiaqi Chen, School of Mathematics & Statistics Teacher's Scholarship;

University of New South Wales: School of Mathematics and Statistics Prizegiving 2009

The School of Mathematics and Statistics Prizegiving was held on Friday 3 April 2009. Congratulations to all the prizewinners:

- School of Mathematics and Statistics Prize 1: for best performance in MATH-1131 Mathematics 1A or MATH1141 Higher Mathematics 1A, and MATH1231 Mathematics 1B or MATH1241 Higher Mathematics 1B in a Bachelor program: Anthony Morris
- School of Mathematics and Statistics Prize 2: for best performance in the core Level 2 Higher Mathematics Units in a Bachelor Program: Chin Pin Wong
- Head of School's Prize: for excellence in four or more mathematics units in Year 2 in a Bachelor Degree Program. Shared by: Stephen Sanchez, Kieran Leong and Timothy Yip
- The C.H. Peck Prize: for best performance in Year 2 Mathematics by a student proceeding to Year 3 in the School of Mathematics and Statistics: Chin Pin Wong
- J.R. Holmes Prize: for excellence in Level 3 Pure Mathematics courses in a Bachelor Degree Program: Michael Abbott
- J.R. Holmes Prize: for excellence in at least four pass-level Pure Mathematics Level 3 Units: Gregory Chung
- Applied Mathematics Prize: for excellence in Level 3 Applied Mathematics courses in a Bachelor Program: Adrian Nicholas
- The Michael Mihailavitch Erihman Award for the best performance in examinations in a mathematics major offered by the School of Mathematics and Statistics in any one year. Shared by: Chin Pin Wong and Anthony Morris
- Department of Employment and Workplace Relations Prize for Statistical Methods in Social and Market Research: Timothy Harrold
- The Alma Douglas prize for Statisitcs for the best performance in Statistics courses in the School of Mathematics and Statisitcs: Gordana Popovic
- Weather Company Prize: for the best performance in MATH3261 Fluids, Oceans and Climate: Matthew Perrett
- Weather Company Prize: for the best performance in MATH2240 Introduction to Oceanography and Metoerology: Ian Coghlan

- The SAS Institute Australia Pty Ltd Prize: for the best performance in MATH-2871 — Data Management for Statistical Analysis: William Yung
- George Szekeres Award for a student entering final year Pure Mathematics Honours: Oliver Nunn
- The Buchwald Award in Applied Mathematics for a student entering the final year of the Honours Course in Applied Mathematics: Matthew Perrett

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Completed PhDs

Monash University

- Dr Gareth Kennedy, *Problems in stellar and planetary dynamics*, supervisor: Dr Rosemary Mardling.
- Dr Hamed Moradi, *Local helioseismology of magnetic activity*, supervisors: Professor Paul Cally and Dr Alina Donea.

University of Ballarat

- Dafik, *Study of graphs close to the Moore bound*, supervisors: Mirka Miller and Joe Ryan.
- Dr Karim Mardaneh, *Information technology and risk management*, supervisors: Adil Bagirov and Musa Mammadov.
- Dr Evgenni Sharikov, *Conditions for global minimum through abstract convexity*, supervisors: Alex Kruger, Sid Morris and David Yost.
- Dr Jakub Teska, *Graphs with bounded degree*, supervisors: Mirka Miller and Joe Ryan.

University of Melbourne

- Norman Do, *Intersection theory on moduli spaces of curves via hyperbolic geometry.* supervisors: Paul Norbury and Craig Hodgson.
- Andrew Downes, *Boundary crossing probabilities for diffusion process and related problems*, supervisor: Kostya Borovkov.
- Maya Muthuswamy, *Micromechanics of force transmission in dense, cohesionless granular assemblies.* supervisor: Antoinette Tordesillas.

University of New South Wales

- Patrick Costello, *The mathematical structure of the BRST constraint method*, supervisor: Hendrik Grundling.

University of Queensland

- Dr Benjamin R. Smith, *Decompositions of generalised complete graphs*, supervisor: Elizabeth Billington, Associate supervisors: Nicholas Cavenagh and Barry Jones.

University of Sydney

- Dr Tony Vassallo, *New models for pricing credit derivatives*, supervisor: Associate Professor Peter Buchen.

Awards and other achievements

Australian National University

- Xu-Jia Wang has been elected to the Australian Academy of Sciences.

Griffith University

Associate Professor Al Gabric and Dr Anand Tularam have won mathematics teaching awards in their school this semester 2009. The best lecturer award went to Al and the best course to Anand as voted by students attending Applied Mathematics Courses at Griffith University, and thus improving the first-year learning experience of mathematics students at the university.

Dr Anand Tularam has won the best lecturer and best course for first-year applied mathematics for a number of years now as he attempts to improve the first-year students beliefs about mathematics. Only now are the first-year students learning to appreciate the importance of mathematical applications to real life especially in the environment/engineering sciences and related courses at Griffith University. This can only be great for mathematics overall as the graduates complete their degrees and gossip about mathematics in terms of how useful and applied it can be.

The mathematics teaching team at Griffith School of Environment have made it their duty to improve the beliefs of first-year and other mathematics students so that they promote the learning of mathematics to their children as a longer term goal in this venture.

Monash University

- Congratulations to Professor Christian Jakob on being awarded another three-year grant from the Department of Energy, Washington DC, in the amount of US$427 000K (A$660K). The research project is entitled 'The relationship between large-scale and convective states in the tropics: towards an improved representation of convection in large-scale models'.
- Congratulations to Professor Michael Reeder on being awarded a research grant of US$316 000 (A$407K) from the Office of Biological and Environmental Research, Department of Energy, Washington DC, for a research project entitled 'The role of gravity waves in the formation and organisation of clouds during TWPICE'.

University of Ballarat

- Dr A. Bagirov and Professor J.L. Yearwood jointly with Professor W. Moran (University of Melbourne) and Dr A.F. Barton (GWM Water), won an ARC

Linkage grant 'Integrating dynamic and optimization models for efficient pipeline system operations in an evolving water and energy market'.

University of Sydney

- In the Queen's Birthday Honours, Associate Professor Terry Gagen has received an AM, for services to higher education.
- Anthony Henderson was awarded a Faculty of Science Citation for Excellence in Teaching.

Appointments, departures and promotions

Monash University

- Dr Leo Lopes commenced 25 May 2009 as Research Fellow in the School of Mathematical Sciences. His research interests are in applied optimisation under uncertainty, and in applied computational optimisation, especially decision support systems.

University of Ballarat

- Dr Prabhu Manyem, Dr Bing Du and Dr Lynne Gleeson have resigned, effective from about the end of Semester 1, 2009.

University of Newcastle

- Dr Con Lozanovski , formerly a Mathematics Research Fellow at Swinburne University of Technology, has taken up a position as a Mathematics Lecturer.
- Dr Jeff Hogan has been appointed as a Senior Lecturer in Mathematics.
- Dr Wadim Zudilin has been appointed as Associate Professor in Mathematics.
- George Willis has been promoted to Professor in Mathematics.

University of New South Wales

- Dr Josef Dick joined the Applied Department as Lecturer in January this year.

University of Queensland

- Dr Hao Yin has been appointed as a Postdoctoral Research Fellow working with Min-Chun Hong on his ARC project 'Geometric partial differential systems and their applications' from 3 August.
- Mr Zdravko Botev has been appointed as an Associate Lecturer from 14 September to 31 December. His research interests include applied statistics, nonparametric kernel density estimation, Monte Carlo simulation of Rare events, Markov Chain Monte Carlo methods, stochastic search algorithms and stochastic differential equations.

- Dr Stephan Tillmann has been appointed Lecturer in Pure Maths commencing 1 July.
- Dr Pee Choon Toh has been appointed as a Research Fellow working with Ole Warnaar from 20 July.
- Dr Gary Griffith has been appointed as a Postdoctoral Research Fellow working with Anthony Richardson/Hugh Possingham commencing 3 August.

University of Southern Queensland

- Dr Trevor Langlands has joined the Department of Mathematics and Computing as a lecturer. He came from UNSW where he was a senior research associate. Trevor's current research interests include modelling anomalous sub-diffusion using continuous time random walk theory and fractional calculus differential operators with applications in nerve signal propagation in spiny dendrites and plaque formation in Alzheimer's Disease.

University of Sydney

- Oya Selma Klanten, research associate with Jean Yang.
- Penghao Wang, research associate with Jean Yang.

University of Western Australia

- Dr John Bamberg has joined the School as a Research Assistant Professor.
- Dr Edward Cripps has joined the School as a Lecturer.
- Dr Steve Su has joined the School as a Lecturer.

New Books

University of New South Wales

Jeyakumar, V. and Luc, D.T. (2008). *Nonsmooth Vector Functions and Continuous Optimization*, Vol. 10, 280 pp. Springer Series on Optimization and its Applications. Springer, New York.

Conferences and Courses

Conferences and courses are listed in order of the first day.

AMSI-ANU Workshop on Spectral Theory and Harmonic Analysis

Date: 13–17 July 2009
Venue: The Australian National University, Canberra
Web: http://wwwmaths.anu.edu.au/events/SpectralTheory09/

Future models for energy and water management under a regulated environment

Date: 20–22 July 2009
Venue: Queensland University of Technology, Brisbane
Web: http://www.amsi.org.au/energy.php

Mini Winter School on Geometry and Physics

Date: 20–22 July 2009
Venue: Institute for Geometry and its Applications, University of Adelaide
Web: http://www.iga.adelaide.edu.au/workshops/winterschool2009.html

Groups St Andrews 2009 in Bath

Date: 1–15 August 2009
Venue: University of Bath, Bath, UK
Web: http://www.groupsstandrews.org/2009/

IMST 2009-FIM XVIII

Date: 2–4 August 2009
Venue: Jaypee University of Information Technology, Waknaghat/ Shimla, India
Web: http://www.juit.ac.in/IMSTFIM2009/imst_fim_2009.htm

Dresden 2009 Conference

Date: 11–17 September 2009
Venue: Dresden, Saxony, Germany
Web: http://www.informatik.htw-dresden.de/~paditz/SecondAnnouncement Dresden2009.doc
Email: alan@rogerson.pol.pl

Third Japanese/Australian workshop on real and complex singularities

Date: 15–18 September 2009
Venue: The University of Sydney (Medical Foundation Auditorium)
Web: http://www.maths.usyd.edu.au:8000/u/laurent/RCSW

AustMS 2009 Early Career Researchers Workshop

Date: 27 September 2009
Venue: Mt Lofty House, Adelaide Hills
Web: http://www.unisa.edu.au/austms2009/earlycareer/
Organisers: Bronwyn Hajek (Bronwyn.Hajek@unisa.edu.au),
Anthony Henderson (anthonyh@maths.usyd.edu.au)

53rd AustMS Annual Conference, 2009

Date: 28 September – 1 October 2009
Venue: University of South Australia, City West Campus, Adelaide
Web: http://www.unisa.edu.au/austms2009/
E-mail: Austms09@unisa.edu.au

The early-bird registration has been extended until Friday 17 July 2009.

Please note the following amendment: On the night of Friday 2 October 2009, the Royal Institution of Australia is organising a Reception at Adelaide Town Hall, featuring a public lecture 'Cosmic Distance Ladder' by Terence Tao, to which the participants of the AustMS 2009 conference are cordially invited. Please extend your travel arrangements accordingly if you plan to attend this event.

9th Engineering Mathematics and Applications Conference (EMAC2009)

Date: 6–9 December 2009
Venue: University of Adelaide, South Australia
Web: http://www.maths.adelaide.edu.au/emac2009/
Contact: Andrew Metcalfe (andrew.metcalfe@adelaide.edu.au)

AMSI workshop: new directions in geometric group theory

Date: 14–18 December 2009
Venue: The University of Queensland, Brisbane
Web: http://sites.google.com/site/ggtbrisbane/

NZIMA/NZMRI Summer Workshop

Date: 3–10 January 2010
Venue: Hanmer Springs, New Zealand
Web: http://www.math.auckland.ac.nz/wiki/2010_NZMRI_Summer_Workshop

AMSI ICE–EM 2010 Summer School

Date: 11 January – 5 February 2010
Venue: La Trobe University
Contact: Grant Cairns, Director (G.Cairns@latrobe.edu.au)

Call for Contributions:
Educational Interfaces between Mathematics and Industry

Date: 19–23 April 2010
Venue: Lisbon, Portugal
Web: http://www.cim.pt/eimi/
Contact: Gail FitzSimons (gail.fitzsimons@education.monash.edu.au)

International Congress of Mathematicians: ICM2010

Date: 19–27 August 2010
Venue: Hyderabad International Convention Centre, Hyderabad, India
Web: www.icm2010.org.in

Preparations for the Congress are now underway. Some information about the city of Hyderabad, pre-registration, registration, some practical information about visiting India, et cetera, can be found at our website. A list of satellite conferences that are being planned is also available at the website.

Detailed instructions for registration, financial aid programs, as well as information on hotel accommodation, list of invited speakers, lecture program, cultural program etc. will be put on the website as and when they are finalised.

On-line pre-registration will start on 15 May 2009 at the ICM 2010 website. It does not involve any payment. The pre-registered participants will be apprised of new developments by e-mail and will receive reminders of upcoming deadlines. Please do pre-register if you intend to participate: it will be of great help to us in our planning the event.

We look forward to your participation at the ICM 2010 in Hyderabad.

There are posters for ICM2010 at http://www.icm2010.org.in/posters.php. Feel free to print them out and display them at appropriate locations.

Visiting mathematicians

Visitors are listed in alphabetical order and details of each visitor are presented in the following format: name of visitor; home institution; dates of visit; principal field of interest; principal host institution; contact for enquiries.

Dr Barbara Brandolini; University of Naples 'Federico II'; 5 to 28 August 2009; mathematical analysis; USN; F. Cirstea

Jose Burillo; Universitat Politècnica de Catalunya, Barcelona, Spain; 20 July 2009 to 20 August 2009; geometric group theory, Richard Thompson's groups, amenability; UQL; Dr Murray Elder

Ms Corrie Jacobien Carstens; University of Amsterdam, The Netherlands; 23 February to 15 August 2009; –; UMB; Dr Craig Westerland

Prof Suhyoung Choi; KAIST, Dept of Mathematical Sciences; 28 February to 31 August 2009; –; UMB; Craig Hodgson

Dr Florica Cirstea; University of Sydney; 14 July 2008 to 14 July 2011; applied and nonlinear analysis; ANU; Neil Trudinger

Dr Robert Clark; University of Wollongong; 1 July 2008 to 1 July 2011; statistical science; ANU; Alan Welsh

Ms Zajj Daugherty; University of Wisconsin; 25 May to 16 August 2009; –; UMB; Prof Arun Ram

Prof Zengji Du; School of Mathematical Sciences, Xuzhou Normal University; 22 October 2008 to 22 October 2009; differential equations; UNSW; Chris Tisdell

Prof David Eisenbud; University of California at Berkeley; 11 to 17 July 2009; algebraic geometry; USN; J.J. Cannon

Prof Michael Finkelberg; Independent University of Moscow; 1 to 30 November 2009; algebra; USN; A.I. Molev

Zhao Guohui; Dalian University of Technology, China; until August 2009; –; CUT

Prof Zongming Guo; Henan Normal University, China; October to December 2009; nonlinear PDE; UNE; Prof Yihong Du

Dr Rainer Hollerbach; Applied Mathematics, Leeds; 9 February to 28 August 2009; nonlinear simulation of magneto-shear tachocline instabilities in the sun; MNU; Prof Paul Cally

Prof Tien-Chung Hu; National Tsing Hua University; 24 February to 31 July 2009; Convergent results for sums of dependent random variables, with possible applications to V-Statistics; USN; N.C. Weber

Mr Alan Huang; –; 1 June to 15 September 2009; statistics; USN; N.C. Weber

Wei Jin (student); Central South University, China; 20 September 2008 to September 2010; –; UWA; Cheryl Praeger

Dr Gwenael Joret; Universite Libre de Bruxelles; 30 March to 24 July 2009; –; UMB; David Wood

Prof Phan Quoc Khanh; International University, Vietnam National University; August 2009; variational analysis; UBR; Alex Kruger

Prof Satoshi Koike; Hyogo University of Teacher Education; 30 August to 30 September 2009; Lipschitz properties of subanalytic sets; USN; L. Paunescu

Mr Gye-Seon Lee; KAIST, Korea; 26 February to 1 September 2009; –; UMB; A/Prof Craig Hodgson

Prof. Zhigui Lin; Yang Zhou University, China; August to September 2009; nonlinear PDE; UNE; Prof Yihong Du

Prof C.C. Lindner; Auburn University, USA; 11–25 July 2009; combinatorics; UQL; Elizabeth Billington

Prof Zhaoli Liu; Capital Normal University, China; July to August 2009; nonlinear PDE; UNE; Prof Yihong Du

Kek Sie Long; University Tun Hussein Onn, Malaysia; May 2009 to November 2009; –; CUT; –

Dr Kamal Khuri-Makdisl; –; 15 August to 15 September 2009; MAGMA; USN; J.J. Cannon

Prof James Meiss; University of Colorado at Boulder; 22 June to 8 August 2009; applied maths; USN; H. Dullin

Hu Ming; Jiangsu University of Science and Technology, Zhenjiang, China; May 2009 to October 2009; –; CUT; –

Prof Christine Mueller; University of Kassel, Germany; 15 October to 24 December 2009; –; UMB; Prof Richard Huggins

Mr Ege Rubak (student); Aalborg Uni, Denmark; April to November 2009; –; UWA; Adrian Baddeley

Mr Gye-Seon Lee; KAIST, Korea; 26 February to 1 September 2009; –; UMB; A/Prof Craig Hodgson

Prof Makato Ozawa; Komazawa University, Japan; 1 April 2009 to 31 March 2011; –; UMB; Prof Hyam Rubinstein

Dr Eric Ragoucy-Aubezon; –; 11 October to 11 November 2009; algebra; USN; A.I. Molev

Mr Ege Rubak (student); Aalborg University, Denmark; April to November 2009; –; UWA; Adrian Baddeley

Dr Leonid Rybnikov; IAS, Princeton University; 1 to 30 November 2009; algebra; USN; A.I. Molev

Dr Alan Stapleton; University of Michigan; 22 June to 21 August 2009; invariant theory, cellularity and geometry; USN; G.I. Lehrer

Dr Damien Stehle; Ecole Normale Superieure, Lyon; 19 July 2008 to 18 July 2009; computational aspects of lattices; USN; J.J. Cannon

Bijan Taeri; Isfashan Uni Technology, Iran; November 2008 to September 2009; –; UWA; Cheryl Praeger

Dr Kilkothur Tamizhmani; Pondicherry University; 9 October to 7 November 2009; applied maths; USN; N. Joshi

Prof Aera Thavaneswaran; –; 1 to 29 August 2009; statistics; USN; M.S. Peiris

Moorum Theeraech; Mahidol University, Thailand; until October 2009; –; CUT; –

Prof Ann Thomas; Cornell University/Oxford University; 10 June to 31 July 2009; geometric group theory; UNSW; Ian Doust

Dr Cipriam A. Tudor; –; 16 to 28 September 2009; statistics; USN; Q. Wang

Dr Helena A. van der Waal; –; 3 June to 30 July 2009; MAGMA; USN; J.J. Cannon

Ms Martha Yip; University of Wisconsin-Madison, USA; 9 March to 20 August 2009; –; UMB; Prof Arun Ram

Dr Zhitao Zhang; Chinese Academy of Science, Beijing; 15 September to 15 November 2009; nonlinear analysis; USN; E.N. Dancer

Prof Huansong Zhou; Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences; October to November 2009; nonlinear PDE; UNE; Prof Yihong Du

AustMS

AustMS Accreditation

The secretary has announced the accreditation of:

- Dr Simon L. Marshall of CSIRO Mathematical and Information Sciences;
- Dr Mark D. McDonnell of the Institute of Telecommunications Research, University of South Australia;
- Dr Steven G. Lord of the School of Mathematical Sciences, the University of Adelaide; and
- Dr Graham T. Clarke of the School of Mathematical & Geospatial Sciences, RMIT,

as Accredited Members (MAustMS).

AustMS Special Interest Meeting Grants: call for applications

Members are reminded that details of funding for Special Interest Meeting Grants are available in the *Gazette* **36**(1), pp. 81–82, from earlier this year. See also http://www.austms.org.au/Publ/Gazette/2009/Mar09/AustMS.pdf for an electronic version of this.

Please email Secretary@austms.org.au for an application form.

Honorary Fellows: call for nominations

In the *Gazette* **33**(1), March 2006, pp. 69–70, the Rules for the Honorary International Fellowship of the Australian Mathematical Society are listed. (See also www.austms.org.au/Publ/Gazette/2006/Mar06/austmsnews.pdf.)

In accordance with Rule 4(a) I hereby call for nominations. These should be sent electronically to Secretary@austms.org.au before the end of August 2009.

Elizabeth J. Billington
AustMS Secretary
E-mail: ejb@maths.uq.edu.au

Elizabeth arrived at the University of Queensland from England in 1971, for a two-year stay; she is still there now, as a Reader/Associate Professor. Besides being Secretary of the Australian Mathematical Society, she is also Editor-in-Chief of the Australasian Journal of Combinatorics. She has also been on the Council of the Combinatorial Mathematics Society of Australasia for the past eleven years. Her research is in combinatorics; she largely works in graph decompositions and combinatorial designs.

The Australian Mathematical Society

President:	Professor Nalini Joshi	School of Mathematics and Statistics The University of Sydney NSW 2006, Australia. n.joshi@usyd.edu.au
Secretary:	Dr E.J. Billington	Department of Mathematics University of Queensland QLD 4072, Australia. ejb@maths.uq.edu.au
Treasurer:	Dr A. Howe	Department of Mathematics Australian National University ACT 0200, Australia. algy.howe@maths.anu.edu.au
Business Manager:	Ms May Truong	Department of Mathematics Australian National University ACT 0200, Australia. office@austms.org.au

Membership and Correspondence

Applications for membership, notices of change of address or title or position, members' subscriptions, correspondence related to accounts, correspondence about the distribution of the Society's publications, and orders for back numbers, should be sent to the Treasurer. All other correspondence should be sent to the Secretary. Membership rates and other details can be found at the Society web site: http://www.austms.org.au.

Local Correspondents

ANU:	J. Cossey
Aust. Catholic Univ.:	B. Franzsen
Aust. Defence Force:	R. Weber
Bond Univ.:	N. de Mestre
Central Queensland Univ.:	R. Stonier
Charles Darwin Univ.:	I. Roberts
Charles Sturt Univ.:	J. Louis
CSIRO:	C. Bengston
Curtin Univ.:	J. Simpson
Deakin Univ.:	L. Batten
Edith Cowan Univ.:	U. Mueller
Flinders Univ.:	R.S. Booth
Griffith Univ.:	A. Tularam
James Cook Univ.:	S. Belward
La Trobe Univ. (Bendigo):	J. Schutz
La Trobe Univ. (Bundoora):	B. Davey
Macquarie Univ.:	R. Street
Monash Univ.:	B. Polster
Murdoch Univ.:	M. Lukas
Queensland Univ. Techn.:	G. Pettet
RMIT Univ.:	Y. Ding
Swinburne Univ. Techn.:	J. Sampson
Univ. Adelaide:	T. Mattner
Univ. Ballarat:	A. Kruger
Univ. Canberra:	P. Vassiliou
Univ. Melbourne:	B. Hughes
Univ. Newcastle:	J. MacDougall
Univ. New England:	I. Bokor
Univ. New South Wales:	C. Tisdell
Univ. Queensland:	H.B. Thompson
Univ. South Australia:	J. Hewitt
Univ. Southern Queensland:	B. Loch
Univ. Sunshine Coast:	P. Dunn
Univ. Sydney:	J. Parkinson
Univ. Tasmania:	B. Gardner
Univ. Technology Sydney:	E. Lidums
Univ. Western Sydney:	R. Ollerton
Univ. Western Australia:	V. Stefanov
Univ. Wollongong:	R. Nillsen
Victoria Univ.:	P. Cerone

Publications

The Journal of the Australian Mathematical Society

Editor: Professor M. Cowling
School of Mathematics
University of Birmingham
Edgbaston, Birmingham B15 2TT
UK

The ANZIAM Journal

Editor: Professor C.E.M. Pearce
School of Mathematical Sciences
The University of Adelaide
SA 5005
Australia

Bulletin of the Australian Mathematical Society

Editor: Associate Professor D. Taylor
Bulletin of the Australian Mathematical Society
School of Mathematics and Statistics
The University of Sydney
NSW 2006
Australia

The *Bulletin of the Australian Mathematical Society* aims at quick publication of original research in all branches of mathematics. Two volumes of three numbers are published annually.

The Australian Mathematical Society Lecture Series

Editor: Professor C. Praeger
School of Mathematics and Statistics
The University of Western Australia
WA 6009
Australia

The lecture series is a series of books, published by Cambridge University Press, containing both research monographs and textbooks suitable for graduate and undergraduate students.

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