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The Australian Mathematical Society

Gazette

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- Classroom notes on presenting mathematics in an elegant way
- Items relevant to mathematics education
- Letters on relevant topical issues
- Information on conferences, particularly those held in Australasia and the region
- Information on recent major mathematical achievements
- Reports on the business and activities of the Society
- Staff changes and visitors in mathematics departments
- News of members of the Australian Mathematical Society

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Volume 35 Number 5 2008

Editorial

Welcome to the final issue of the *Gazette* for 2008.

With this issue Peter Hall contributes his final column as President of the Society. Along with all members we would like to thank Peter for the excellent work he has done as President, and in particular we thank him for his support and encouragement during our first two years working on the *Gazette*. It has been a pleasure to work with him, and we look forward to working with the incoming President, Nalini Joshi.

We also hear from a previous President of the Society, Bob Anderssen, in his contribution to our regular Maths Matters series. Bob explains explains the importance of changing the public image of the mathematics profession, by highlighting the importance of mathematical training in the successful careers of all maths graduates. Bob argues this would have a positive effect for mathematics not just within the university system, but also in its perception by the general public. We're sure Bob's column will stimulate vigorous discussion and we'd be very happy to hear reader's thoughts on this, or any other articles published in the *Gazette*.

This issue introduces a new feature to the *Gazette*, Bill Blyth's regular column on the Access Grid. This technology is now actively supported by many Australian universities, and by organisations such as AMSI. In his first column Bill introduces the concept of the Access Grid and gives an overview of its use in mathematics for collaborative research and teaching. We hope the column will help to promote this new resource for our geographically spread maths community.

Phil Broadbridge writes about a particularly timely topic in his AMSI News: the important role mathematics has to play in ensuring economic stability. It is very enlightening to read about AMSI's recent involvement in reviewing the mathematics used by the Australian Prudential Regulation Authority given our new understanding of the risks taken in the global financial markets.

We are very pleased to include in this issue a paper by Neil Saunders; when he presented the paper at the Annual Meeting in September 2007 he was the joint winner of the B.H. Neumann Prize for best student talk. The issue also includes technical papers by Ovidiu Furdui and John Miller.

Also in this issue, Nev Fowkes and Phil Broadbridge report on a highly successful graduate modelling workshop in Canada and we have a short report on the conference on mathematics and computers in sport held in Tweed Heads from Neville de Mestre. Peter Johnston has contributed his annual report on higher degree and honours maths and statistics completions for 2007. And of course there are book reviews, Norman Do's Puzzle Corner (congratulations to Gerry Myerson, winner of the book voucher for Puzzle Corner 8!), as well as all the news from mathematics around Australia.

Finally we'd like to once again thank all the *Gazette* readers for your continued support, and particularly for your excellent contributions. We wish you all a very happy festive season, and all the best for the new year!

Happy reading from the *Gazette* team.

President's column

Peter Hall*

I would like to use this brief column, my final one as President, to thank some of the many people who have assisted me, or helped Australian mathematics, during the last 26 months. First I must thank Liz Billington, who as Secretary of the Society is the person responsible for keeping things moving. Without her remarkable vigilance, insight and attention to detail, and the continuity and corporate memory that she provides through the tenure of many presidents, our operations would quickly unravel.

Our Treasurer, Algy Howe, has helped guide us through turbulent financial times, and supported initiatives to place some of our reserves into the care of a major Australian university, for investment at a rate of return that we could not ensure on our own. At the time of writing, the details of this proposal were still being worked out, but the Steering Committee has already had discussions with the university's financial officer and been impressed by the opportunities.

Birgit Loch and Rachel Thomas, editors of the *Gazette*, have helped disseminate news throughout my term. Birgit had sole responsibility for the *Gazette* while Rachel was on maternity leave. This period coincided with substantial upheaval and trauma at the University of Southern Queensland, where Birgit works, and she must have wondered on many occasions how she could possibly keep the *Gazette* on track. But on track it stayed. We're very grateful to you, Birgit.

The Canberra office, and most of all May Truong, worked hard to ensure that the introduction of our new website was as smooth and as timely as possible. Thanks to May's efforts you should be able to pay your 2009 subscriptions online, rather than by mail. I owe a substantial debt to Ross Moore, too, for all his efforts in this regard. Without his expert knowledge, and the considerable time he spent liaising with our website designer, the design would have been both less effective and less timely.

Our journal editors, Michael Cowling (the *Journal*), Charles Pearce (the *ANZIAM Journal*) and Don Taylor (the *Bulletin*), worked hard to make the transfer of publication to Cambridge University Press as trouble-free as possible. I should add that Michael also played a leading role in this development before I became president, and so the smoothness of the transition owes a great deal to his input over several years.

The CUP contacts made by Cheryl Praeger, editor of the Society's *Lecture Series*, which has been published by CUP for some time, also helped the transition. I'm grateful too for Cheryl raising the profile of Australian mathematics through her work on the IMU Executive Committee. The IMU intervened very constructively

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in our dispute with USQ over the fate of that university's mathematics program. The intervention was of substantial benefit; for example, it gave the dispute a higher media profile.

The ANZIAM Chairs, Peter Taylor up until last February and subsequently Phil Howlett, have been particularly supportive. I especially remember the help that ANZIAM gave during the journal-ranking debate, drawing up a comprehensive comparison of the previous and the ARC rankings (here Jim Denier deserves special thanks) and helping to ensure that our communications with members around the country, and with the ARC, were as smooth and constructive as possible.

The Society's Steering Committee and Council, particularly Vice-Presidents Mathai Varghese and Ole Waarner, have also been very helpful with advice and assurance. Sometimes we've called on them for quick comments or approval, and they have always risen to the occasion. Mathai has worked tirelessly to ensure that our conferences, and the Mahler Lecture series, run smoothly.

Many other members of the Society, not just those who are members of the Society's Executive or our committees, have given sterling service to Australian mathematics during the last two years. If I may I'd like to single out Hyam Rubinstein, Terry Tao and Jan Thomas for special mention. Hyam, Chair of the National Committee for the Mathematical Sciences, and Jan, AMSI's Executive Officer, have been responsible for more than a few submissions to the federal government's many reviews of the higher education sector, and also for submissions to other reviews relating to mathematics in Australia. (I gather that there are currently some 140 reviews, initiated by the Rudd government, under way.) Hyam also played a leading role during the journal-ranking debate, calling together the National Committee's subcommittees to oversee the re-ranking process. Terry Tao gave, and continues to give, extraordinary assistance to us all, not just in the context of the dispute with USQ but much more generally. His capacity for working fast, accurately and constructively, at a high level and in large quantity, still astounds me.

I could mention many others. However, Birgit and Rachel would not thank me for taking up any more space in the *Gazette*, which under their editorship is receiving many submissions. Therefore let me close with a simple 'welcome' to Nalini Joshi, who will assume the Presidency in December and who will, I'm sure, take the Society to greater heights during the next two years. Nalini's many talents, particularly her tremendous capacity for organisation and for cutting to the heart of any issue, will be of great benefit to us all. Many challenges lie ahead, many of them unpredictable, before the mathematical sciences can be restored to their rightful position of strength in Australia.

Peter Hall is a statistician, with interests in a variety of areas of science and technology (particularly the physical sciences and engineering). He got his first degree from The University of Sydney in 1974, his MSc from The Australian National University in 1976, and his DPhil from University of Oxford in the same year. Peter is interested in a wide variety of things, from current affairs to railways and cats.

It is the perception that counts, not the reality!

Bob Anderssen*

My parents would not countenance me going to university to simply study mathematics. For them, they had no doubts. Mathematics and science were the corner stones of knowledge. 'But, Robert, if you only study mathematics and science, you will never get a (good) job.' They pointed to the daughter of close friends who had studied science and now worked as a chemist in a jam factory.

Initially, I studied engineering before switching to science and then to mathematics.

Naturally, my parents, like all good parents, wanted to maximise the opportunities for their children, and were willing to make the necessary sacrifices. Given their and their friends' backgrounds and the time when they grew up, they had a perception that was not reality. Just reflect for a moment on the family dynamics that must have occurred in situations where one of the children wanted to be an artist, a write, a poet, an actor.

The basic perception still persists — mathematics is important, fundamental, special, but not essential to guaranteeing a good job. For me, the challenge for the mathematics profession is to turn this around so that the perception becomes 'mathematics is essential to guaranteeing a good and secure job', no matter what might be the chosen profession. This has always been true, but its reality is not widely understood or appreciated. Interestingly, from a community perspective, it is accepted that engineers study mathematics, while for other professions, such as chemistry and biology, mathematics is not involved.

In my view, engineers, chemists, biologists and others are utilising brain circuitry similar to that used by mathematicians when involved with creative activities like planning an experiment, modelling or pattern recognition. This is a matter which will be illustrated and pursued below. The scientific understanding of how the brain works has progressed to the point where it is now appreciated that, in one way or another, similar circuitry in the brain is being used for quite different activities. The most widely know example is the high that one gets during long distance running or jogging, and as the initial effect induced by a recreational drug.

For myself and other colleagues, this false perception about mathematics is the result of a general lack of understanding in the wider community about the role, importance and practice of mathematics in everyday applications, such as tomography (based on the mathematics of Johann Radon), GPS and TV. Consequently,

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my overall goal is to give some examples of why the role and importance of mathematics is not as widely appreciated as it should be, to make some suggestions as to how this can be changed, to stress the growing importance of mathematical modelling and to make some comments about challenges that will impact on the future development of mathematics.

Why does this false perception exist?

In collaborative research, the biology (engineering, industrial, or other) colleague is only interested in the results that the mathematician collaborator generates to answer the matter under examination. The technical mathematical details by which the results have eventually been derived are, at most, of marginal interest, even though they give the mathematicians involved a feeling of enhanced self-esteem or result in new insight about mathematics itself. The fact that the results could be derived and validated mathematically in different ways, might have a wider applicability or may be of interest mathematically in their own right is only of interest to the mathematicians.

For example, in cereal science, the plant breeder wants simple techniques, such as NIR (near infrared) spectroscopy, to distinguish between wheat varieties that make good breads, pastas, cakes or biscuits. The breeder is not interested in any of the technical details about the calibration-and-prediction protocols used to define how to segregate the wheats on the basis of their NIR spectra, or some other experimental protocol. The only proof required is validation that the segregation is consistent with the breeders', growers', millers' and customers' expectations. It is interesting how, in such situations, the proof (validation) is not the mathematical details but the response and approval from higher authorities. Painful as it might be for the mathematician or statistician, this is the reality of the situation. It is here that our perception is incorrect — though mathematics has played a crucial role, the validation and acceptance comes from the higher authority related to the problem context.

In the study of pattern formation in plants, the biologist likes to reference publications about the reaction-diffusion modelling of pattern formation without appreciating that such models only see the macroscopic structure of the signalling, communication and switching within and between the particular set of cells of the plant involved with the patterning. The relevant comment that summarises the challenge that underlies the utilisation of mathematical models to solve biological problems is that of Willard Gibbs:

> One of the principal objectives of theoretical research in any department of knowledge is to find the point of view from which the subject appears in its greatest simplicity.

In a way, the challenge of finding the appropriate 'point of view' represents a good way to explain and illustrate to a wider audience why mathematical modelling is not as easy and as simple as it is often seen and sometimes lampooned.

Another possible explanation for the false perception that mathematics is not essential in getting a good job is the widespread lack of appreciation of the role and importance of basic mathematical knowledge. A good illustrative example of the importance of basic mathematics comes from the acute sensitivity that occurs regularly in real-world situations such as in exploration geophysics, as is given by the Railroad Rail Problem [1, pp. 3–4 and Chapter 2]. This is a good example where the derivation of a rigorous estimate is quite involved technically, whereas an indicative estimate based on a right-angle triangle approximation gives insight about the apparent intuitive contradictory nature of the solution.

Rules-of-thumb play a key role in industrial decision making on a daily basis. Scientific facts and mathematical/statistical modelling have played and will continue to play a crucial role in their formulation and in determining their domain of validity. Their importance economically should never be underestimated — they allow quick representative decisions to be made which reduces the risk of disaster in challenging situations. They represent an excellent way of motivating the interest of students in the role played by mathematics in applications. However, it is important for all of us to remember and acknowledge that they are based on modelling. The following observation, attributed to George Box [2, p. 424], is an important quotation to remember:

> All models are wrong, but some are useful.

If mathematics is used to solve a problem and fails to give a satisfactory answer, the mathematics is often the only thing that is called into question. The flaw often relates to the assumptions on which the modelling is based, rather than the subsequent mathematical manipulations.

Finally, there is a failure to make people proud of the mathematics that they can do. A common comment from non-mathematical friends, acquaintances and colleagues is that they are 'not good at mathematics'. (A good example of this response is discussed in the recent *Gazette* article by Peter Pleasants [4, p. 90].) However, when the matter is pursued, one finds that it is the last mathematics that has been learnt that is the reason for the claim. The fact that they can solve simple algebraic and geometric problems, or they know and understand Pythagoras' theorem, or can differentiate algebraic relationships, or solve ordinary differential equations is taken for granted. We all find it challenging to learn new mathematics. The higher the level we learn mathematics, the more mathematics we will know intuitively with skill and confidence. It is important to make this fact clear at all stages in the learning of mathematics from pre-school, to primary, to secondary school, to university, and in the daily discussion of quantitative concepts. Some suggestions, on how this might be done, include:

- Guessing: treat the mistake by a student or a colleague as a good guess and build on that.
- Mathematics should not be competitive but cultural. We all enjoy activities like music, solving puzzles, jokes, paintings and gardening. Mathematics should be treated as a similar cultural pursuit in terms of a search for

patterns. A good example is the current popularity of Sudoku and the historic and continuing popularity with crossword puzzles, chess, jigsaw puzzles, etc.

- Understand why others see things differently. Take a rectangular sheet of paper and show young children that it can be folded into two different cylinders, and then ask them which contains the larger volume. The answer is often that they are the same because the areas are the same. For their age and level of mental development, it is an excellent guess. Understanding the reasons why an incorrect answer is given will help make mathematics to be viewed more as a cultural, rather than a competitive, activity.
- The importance of mistakes is that they represent progress. George Polya is reported to have made something like the following comment to a student who thought that George must never make mistakes: 'I regularly make mistakes. The difference between you and me is that I find my mistakes much faster than you find yours.'

An easy way to make people feel self-esteem about their mathematical knowledge is to use illustrative examples of the role of that mathematics in applications. Such advice is hidden in the following comment in the Preface of the book *Methods of Mathematical Physics* by Jeffreys and Jeffreys [3]:

> We think that many students whose interests are mainly in applications have difficulty in following abstract arguments, not on account of incapacity, but because they need to 'see the point' before their interest can be aroused.

In fact, in all levels of research, the challenge and struggle is to 'see the point' that allows new patterns to be identified and explained.

The importance of modelling: the impact of applications on mathematics

> The profound study of nature is the most fertile source of mathematical discoveries. J.P.J. Fourier

In my view, one of the key doorways to stimulating a greater awareness of the reality of mathematics is modelling. It is the cornerstone on which collaboration is based. The model identifies the question the non-mathematical colleague is investigating and to which mathematics can be applied. It is the structure that the mathematician solves, and is a framework from within which an answer is generated and to which the colleague then gives an interpretation.

The same model and solution can have arisen in a completely different situation. The only difference between these two situations is the interpretation that is applied to the solution. The diffusion equation is an excellent example. Its solutions can answer questions about heat conduction, the uptake of nutrients by plants, the infiltration of water into the soil, etc.

It is crucial to understand that much successful modelling is being performed without the assistance of a mathematician or statistician, and in many situations the non-mathematician is unaware that modelling is being implicitly performed. But, in some form or other, mathematics is being utilised.

Frank de Hoog, in his Maths Matters contribution [5] reminds the reader of the fact that

> . . . only a tiny proportion of all the application of mathematics is performed by those who consider themselves to be a mathematician . . . even if we limit ourselves to sophisticated applications of mathematics.

One can surmise that, in such situations, the biologist (engineer, economist, etc.) is using similar circuitry in the brain to that used by the mathematician. Consequently, many of the tasks being performed on a daily basis by the bulk of the community might have a greater neural affinity with mathematics than is currently appreciated. It is therefore an aspect that might assist people in the community to have a greater self-esteem about their problem-solving abilities. There are many smart people out there in the community who are not mathematicians, but are using mathematical skills. Some, such as engineers and computer scientists, have strong mathematical backgrounds and are directly exploiting the circuitry in the brain where that knowledge is stored. Others, like the biologist, are exploiting implicitly the circuitry that evolution has built into our brains to perform (quantitative) pattern recognition tasks.

If one believes, as I do, that the essence of mathematical creativity is seeing new patterns, then many people, such as experimentalists and artists, are using mathematical-related circuitry in their brains.

These ideas are not new!

However, it is a classical situation where the 'obvious must be said over and over again before it is fully understood'. An example of where obvious things must be said over and over again before they are fully understood is global warming. The scientific reality of the increasing average-temperature-evidence was there but not the perception. Now that the perception is there, it is interesting how rapidly community attitudes are changing. Consequently, there is hope for changing the community's perception about mathematics.

In his Math Matters contribution, John Henstridge [6] explained with considerable skill how academic mathematics, and the mathematics profession in Australia, has become a captive of the community's perception that mathematics is important but not essential for the education of professionals. Among other good points, John stresses how important marketing is to the survival of his consulting company. I am in full agreement with his conclusion that the mathematics community must '. . . recognise and support all our graduates and all the work that they do' and that 'It will be a challenge [for the mathematical community] to do so with one voice . . . '.

In the 'Mathematics graduates are highly employable' brochure that the Australian Mathematical Society published and circulated in the mid-1980s and early 1990s, one finds the following quotation by Ross Gittins, Economics Editor, Sydney

Morning Herald:

> . . . if possible do Maths . . . (it) is the single most useful ability to have in your kit-bag to equip you for any eventuality.
>
> In any case employers set a lot of store by mathematical ability and are more likely to hire someone with a good background in Mathematics.

As a profession we must strongly market this point of view. Not only must we do it, even more importantly we must find champions like Ross Gittins who will also do it. We must learn to take a back seat as far as receiving credit for the idea. We must not market it as our idea but the idea of various champions.

Academic perceptions

It is ironic that, when universities need to save funds, they prune mathematics, the least expensive of the disciplines. Money could be saved by pruning the more expensive disciplines. Why don't they? Those more expensive disciplines have strong lobby groups and the perception is that they are essential for the image of the university. The strongest disciplines in this regard are the one who are perceived to be able to guarantee jobs for their graduates.

For me, it is unbelievable that some mathematicians try to justify their existence on the basis that their work might be important in two hundred years. Surely, a wiser, more understanding and less superior attitude would be that they are contributing to the body of mathematical knowledge that is used to solve practical problems, and that the greatest pleasure would be to learn that a recent contribution is already being utilised.

At the ICIAM Congress in 1995 in Hamburg, V.I. Arnold, in his plenary talk, commented in the following manner, exploiting the duality in meaning of both 'pure' and 'applied'

> My best pure mathematics was done in applied mathematics and my best applied mathematics was done in pure mathematics.

Future challenges

I believe many changes are happening now that will have a strong influence on how the future will unfold for the mathematics disciplines and professions:

- Applications will have an increasingly important impact on how mathematics is pursued and funded in the future. A good example is the embracing of biology by various mathematical disciplines when seeking research funding, such as algebraic biology initiatives.
- The great success of modern mathematics is the huge spectrum of results that it has formalised which are available for the solution of practical problems. Because of this pool of knowledge, there is an increasing opportunity and speed with which mathematics can now be found which resolves many new practical problems.

- Practitioners, in collaborations with mathematicians, will increasingly demand relevant and useful interpretations of the mathematics that is being proposed to solve their practical problem. The users of mathematics will become increasingly sceptical of complex mathematical constructs which appeal to the mathematician but tell them little about the processes supposedly being investigated.
- There will be a greater use of computers to perform complex calculations the results of which will be used to make decisions that will impact on our daily lives. The growing importance of preventive medicine is a positive illustration, if the solution of the underlying inverse problem has been solved with sufficient accuracy.
- The great success of technology in the last decade or so has been the development of computer-controlled instrumentation. An obvious daily example is the bar-coding of goods and commodities for electronic checkout. An important scientific example is the various instruments designed to perform DNA sequencing. However, the plethora of data that such instruments are generating, is spawning a need for new paradigms for their analysis and interpretation.
- With the wider use of computers to perform more and more basic tasks, the stage is set for major cultural changes. In this regard, the computer and, hence, the underlying mathematics have a crucial role to play in exploring computationally various scenarios for the solution of global warming challenges, the efficient management of water and petroleum resources and the protection of biodiversity. The use of the computer to first explore alternatives for the solution of such problems will generate an increasing demand for scientists, engineers, biologists, etc. with strong mathematical backgrounds.

In one way or another, the above items will be responsible for more and more people wanting to perform their own mathematical modelling and simulation. This is a clear target group for the mathematics professions to encourage, mentor and support. The technology is there for more people to be owners of their mathematical modelling and simulations. They are not going to stop having such ownership simply because some mathematician or statistician says that they do not have the sophistication to do so. What must be done is to give them the confidence to turn to mathematics professionals for enhancing their mathematical expertise and viewing them as potential collaborators.

Conclusions

As already mentioned above, the professions that have the greatest clout politically and the strongest impact on the community and potential students are the ones that can guarantee (good) jobs for their graduates. This fact plays a key role in behind-the-scenes negotiations and decisions about future directions and funding in academia and the allocation of research funds. The position for the mathematical sciences in such negotiations will be considerably improved once we, speaking with one voice, convince parents, teachers, mentors and students of the new reality that

> . . . mathematical knowledge and skill has been, is and will continue to be essential to guaranteeing a good and secure job in whatever profession a person decides to pursue.

This, in my view, will only be achieved once there is a greater appreciation in society of the role and importance of mathematics at all levels of daily activities. The challenge is to reverse the situation encapsulated in Edward E. David's comment that 'the importance of mathematics is not self-evident'. Reverse it by creating a situation where all become aware of the role of mathematics in their daily activities no matter how humble they are.

The responsibility for doing this rests not only with the mathematical sciences professionals like you and me, but also with creating situations where champions of the non-mathematician community, like journalists, media professionals, and TV presenters and stars, trumpet this new reality loud and clear.

For me, for example, it will be clear that mathematics has become a more essential part of our cultural heritage when the TV weather commentator says something about the mathematics that sits behind the changing temperature and pressure dynamics that are displayed each evening, perhaps explaining that close contour lines in atmospheric pressure imply that the winds there will have a higher velocity than elsewhere.

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[6] Henstridge, J. (2006). Maths matters: If math matters, why don't we see more of it? *Gaz. Aust. Math. Soc.* **33,** 231.

Bob Anderssen was born in Brisbane on January 13, 1939. He completed BSc and MSc degrees, which tended to focus on applied and computational mathematics, at the University of Queensland. His PhD in mathematics, on the numerical performance of variational methods, was obtained at the University of Adelaide, where he was also a tutor in mathematics. In 2008, he received a honorary DSc (honoris causa) from La Trobe University. He has been president of the Australian Mathematical Society and chair of the National Committee for Mathematics. He has been awarded the George Szekeres and Joe Moyal medals, given the G.S. Watson Annual Lecture at La Trobe University in Bendigo and is a Fellow of the Australian Mathematics Society. His hobbies include gardening, hiking and classical music.

Puzzle corner

Norman Do*

Welcome to the Australian Mathematical Society *Gazette*'s Puzzle Corner. Each issue will include a handful of entertaining puzzles for adventurous readers to try. The puzzles cover a range of difficulties, come from a variety of topics, and require a minimum of mathematical prerequisites to be solved. And should you happen to be ingenious enough to solve one of them, then the first thing you should do is send your solution to us.

In each Puzzle Corner, the reader with the best submission will receive a book voucher to the value of $50, not to mention fame, glory and unlimited bragging rights! Entries are judged on the following criteria, in decreasing order of importance: accuracy, elegance, difficulty, and the number of correct solutions submitted. Please note that the judge's decision — that is, my decision — is absolutely final. Please e-mail solutions to N.Do@ms.unimelb.edu.au or send paper entries to: Gazette of the AustMS, Birgit Loch, Department of Mathematics and Computing, University of Southern Queensland, Toowoomba, Qld 4350, Australia.

The deadline for submission of solutions for Puzzle Corner 10 is 1 January 2009. The solutions to Puzzle Corner 10 will appear in Puzzle Corner 12 in the May 2009 issue of the *Gazette*.

Chopping a chessboard

A regular 8×8 chessboard is cut into n rectangles along its gridlines so that no two of the rectangles are congruent. What is the maximum possible value of n?

Photo: S. Schleicher

How many numbers?

Some real numbers are written on a blackboard. It is known that the sum of the numbers is 20, the smallest sum using three of the numbers is 5 and the largest sum using three of the numbers is 7. How many numbers are there on the blackboard?

Penny in a corner

A disk of radius 1 moves so that its circumference is continually in contact with the xy-plane, the yz-plane, and the zx-plane in three-dimensional space. Find the locus of the centre of the disk.

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Complete the table

Below is part of a table with infinitely many rows and infinitely many columns. The entry in the top left corner is 0 and every other square is labelled with the smallest non-negative integer which does not appear in that row to the left of it or in that column above it.

0	1	2	3	4	5	
1	0	3	2	5	4	
2	3	0	1	6	7	
3	2	1	0	7	6	
4	5	6	7	0	1	
5	4	7	6	1	0	

By completing the table, or otherwise, determine which number occurs in the 123rd column and the 456th row.

The way to Heaven

You arrive at a fork in the road. One path leads to the village named Heaven while the other leads to the village named Hell, but you do not know which is which. Consider the following three scenarios.


Photo: L. Emerson

(1) Standing before you are two people. One always tells the truth and one always lies, but you do not know which is which. How can you discover the way to Heaven by asking only one yes-or-no question to one of the two?

(2) Standing before you are two people. One always tells the truth and one always lies, but you do not know which is which. They both understand English but respond using their native language. Fortunately, you remember that the words for YES and NO in their native language are GEB and MIU, but you seem to have forgotten which is which. How can you discover the way to Heaven by asking only one yes-or-no question to one of the two?

(3) Standing before you are three people. One always tells the truth, one always lies and one answers yes-or-no questions randomly, but you do not know which is which. How can you discover the way to Heaven by asking only two yes-or-no questions, each directed to just one of the three?

Solutions to Puzzle Corner 8

The \$50 book voucher for the best submission to Puzzle Corner 8 is awarded to Gerry Myerson.

Watchful wombats

Solution by Ivan Guo: We will prove the statement by induction on n, the number of wombats. The base case of $n = 1$ is trivial since a lone wombat cannot be watched. Suppose that the statement is true for $n = k$ and consider now the case $n = k+2$. Suppose that the wombats A and B have the smallest distance between them and note that they must be watching each other. Now remove A and B from the group. If any of the remaining wombats were watching A or B, let them now turn to watch the closest other wombat in the remaining group. In this new group of size k, each wombat is once again watching its closest neighbour. So by the inductive hypothesis, one of them — say X — is not being watched. Now rewind back to the days when A and B were still around. Wombat X could not have been watched by anyone from the group of size k. Furthermore, X was definitely not watched by A or B, who were too busy watching each other. Therefore, we have proven the statement by induction for all odd positive integers n.

Rubik's cube

Solution by James McCoy: Let the numbers on the faces of the cube be a, b, c, d, e, f, where a is opposite b, c is opposite d, and e is opposite f. Then the products assigned to the vertices are ace, acf, ade, adf, bce, bcf, bde, bdf and Rubik has noticed that

$$ace + acf + ade + adf + bce + bcf + bde + bdf = 2008.$$

Conveniently, this equation can be equivalently expressed in the factorised form

$$(a + b)(c + d)(e + f) = 2008.$$

Now $2008 = 2^3 \times 251$ and each of $a+b$, $c+d$, $e+f$ is at least 2. So the possibilities for the factors $a+b$, $c+d$, $e+f$, up to ordering, are $2, 4, 251$ or $2, 2, 502$. Therefore, the sum of the numbers can be $2 + 4 + 251 = 257$ or $2 + 2 + 502 = 506$.

Crowded subsets

Solution by Iwantme Voucher[!]: A crowded set $A \subseteq S$ is uniquely determined by its common difference d and its smallest element a. By extending the set $\{a, a + d, a + 2d, \ldots\}$ as far as possible, one obtains a crowded set if and only if $a \leq d$ and $a + d \leq n$. Thus, it suffices to count pairs of positive integers (a, d) with $a \leq d \leq n - a$. For a fixed value of a, there are $n - 2a + 1$ such pairs if $2a \leq n$, and none otherwise. Therefore, the total number of crowded subsets of S is

$$\sum_{a=1}^{\lfloor n/2 \rfloor} n - 2a + 1 = \begin{cases} 1 + 3 + 5 + \cdots + (n-1) = n^2/4 & \text{if } n \text{ is even,} \\ 2 + 4 + 6 + \cdots + (n-1) = (n^2-1)/4 & \text{if } n \text{ is odd.} \end{cases}$$

Clock confusion

Solution by Paul Emanuel: Let the angle of the hour hand past noon be α and let the angle of the minute hand past the hour be β. Furthermore, let $\alpha = \alpha_0 + (k\pi)/6$ and $\beta = \beta_0 + (\ell\pi)/6$, where $\alpha_0, \beta_0 \in [0, \pi/6)$ and $k, l \in \{0, 1, 2, \ldots, 11\}$. Then a time is indistinguishable if the proportion of the hour hand past some hour equals the proportion of the minute hand through the hour, and the proportion of the minute hand past some hour equals the proportion of the hour hand through the hour. In other words, we require the following two equations to be true.

$$\alpha_0 \div \frac{\pi}{6} = \frac{6\alpha}{\pi} - k = \frac{\beta}{2\pi}$$
$$\beta_0 \div \frac{\pi}{6} = \frac{6\beta}{\pi} - \ell = \frac{\alpha}{2\pi}.$$

Solving these equations simultaneously leads to

$$\alpha = \frac{2\pi}{143}(12k + \ell) \quad \text{and} \quad \beta = \frac{2\pi}{143}(k + 12\ell).$$

When $k = \ell$, we obtain $\alpha = \beta$ and the hands are overlapping, in which case the time is not indistinguishable. For all other $k, \ell \in \{0, 1, 2, \ldots, 11\}$, α and β are not multiples of $\pi/6$, and so for any such fixed pair (k, ℓ) there is exactly one indistinguishable time. Therefore, there are $12 \times 12 - 12 = 132$ such occasions.

Alternative solution based on work submitted by Sam Krass: Consider a third hand which moves 12 times as fast as the minute hand. The ambiguous times correspond precisely to when the hour hand and third hand coincide. In the 12-hour period between noon and midnight, the third hand travels around the clock 144 times while the hour hand travels around the clock once. Therefore, they coincide precisely 143 times, but 11 of these are when the hour and minute hands are overlapping. Therefore, the number times which are indistinguishable is $143 - 11 = 132$.

Give and Take

Solution by David Angell: Give can choose 16 handfuls of 6 coins and 1 handful of 4 coins. Since there are 17 handfuls altogether, the game finishes as soon as one player has received 9 handfuls, comprising 52 or 54 coins. The other player receives the remaining 48 or 46 coins, so Give can be sure of getting at least 46 coins.

Alternatively, Take can adopt the strategy of accepting any handful of 6 or more coins and refusing any handful of 5 or fewer coins. If the game finishes with Take accepting 9 handfuls, then Take receives at least 54 coins and Give receives at most 46. However, if the game finishes otherwise, then Give receives at most 9 handfuls of at most 5 coins — at most 45 coins in total. Therefore, we conclude that Give can be sure of getting at least 46 coins, but not more.

Playing with polynomials

Solution by Gerry Myerson:

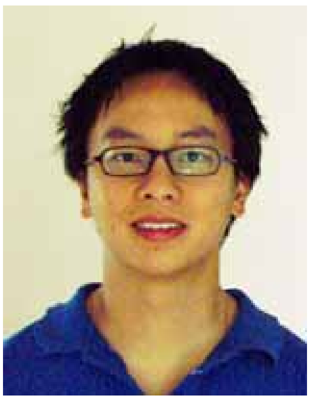
(1) The value of the quadratic polynomial at $x = -1$ is initially -98 and finally 100. Furthermore, each change to its coefficients altered the value at $x = -1$ by ± 1. Therefore, at some point in time the value of the polynomial at $x = -1$ must have been 0. So there was a point in time when the polynomial had -1 as a root. So, by virtue of being a monic quadratic polynomial with integer coefficients, there was a point in time when the polynomial had integer roots.

(2) In theory, one question suffices. Since π is transcendental, there cannot be two distinct polynomials with integer coefficients that take on the same value at π. Thus, $P(x)$ is uniquely determined by the value of $P(\pi)$.
In a somewhat more practical sense, two questions suffice. Ask for $P(1)$ and suppose that the answer is m. Then ask for $P(b)$ where b is an integer greater than m. Then the coefficients of $P(x)$ can be read off from the base-b representation of $P(b)$. This is true because $P(1) < b$ implies that every coefficient of P is less than b, and base b representation of a positive integer is unique.

(3) Take the polynomial

$$F(x,y) = \frac{(x+y)(x+y+1)}{2} + x.$$

Consider the sequence of triangular numbers $0, 1, 3, 6, 10, \ldots$ given by $T_m = (m(m+1))/2$ for $m = 0, 1, 2, \ldots$. Then for every non-negative integer n there is a unique non-negative integer s such that $T_s \leq n < T_{s+1}$. Since $T_{s+1} - T_s = s + 1$, it follows that every non-negative integer can be written uniquely as $n = T_s + r$ with $0 \leq r \leq s$. Now $T_s + r = F(r, s - r)$, and the one-to-one correspondence is clear.



Norman is a PhD student in the Department of Mathematics and Statistics at The University of Melbourne. His research is in geometry and topology, with a particular emphasis on the study of moduli spaces of algebraic curves.

What is the Access Grid? . . . and what is it good for?

Bill Blyth*

A simple question: what is the Access Grid? A simple (but wrong) answer is that the Access Grid is video-conferencing. A better answer is that the Access Grid (AG) is video-conferencing on steroids. However the AG provides many more features and functionality than video-conferencing and might be better referred to as *video-conferencingPlus*. Many commercial video-conferencing products are available and being used increasingly in business, industry and universities. Basic video-conferencing provides video and audio for meetings. This can be augmented by using a document camera to provide a video image of documents to remote locations. On the other hand, many video-conferencingPlus systems now provide extra features, particularly data transmission or some form of desktop sharing which incudes the capability to not only display, but also to remotely control, the software and data analysis. For example, a spreadsheet or Maple file (or any other software file) can be displayed and remotely controlled. One such system is Bridgit, from SMART Technologies who also developed SMART Boards (see http://www2.smarttech.com/st/en-US/Products/Bridgit/Features.htm). Bridgit is being used for undergraduate mathematics teaching, in suitably equipped lecture theatres, across several campuses at Charles Sturt University. Another system, Elluminate, is being used for international remote teaching via desktops for some of the UK's Open University mathematics courses.

The Access Grid was developed at Argonne National Laboratories in the US and further developments have been an international effort, notably from the US, Canada, UK and Australia (see http://www.accessgrid.org). The AG is at the high end of video-conferencingPlus systems, and it's open source (that is, the software's free). The AG is fully featured and flexible: there isn't a typical Access Grid Room (AGR). However an AGR usually has three (or four) screens which are linked and so operate as one large projection screen, referred to as the wall. In addition, usually there are three cameras positioned to capture the presenter and participants from various angles which provide video streams and there is one audio stream.

The wall is often a large projection 'screen' where the three screens are linked into one seamless screen, such as the AGRs at the University of Birmingham and at La Trobe University. In these cases, provision for handwriting is made by using a normal whiteboard (on a side wall) and the Mimeo hardware and software: this

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involves using special magnetic sleeves for each whiteboard marker (with different colours) which are tracked by the Mimeo hardware to produce a digital image of the whiteboard handwriting and sketches.

Bill Blyth in the RMIT University Access Grid Room

The wall can include interactive whiteboards. For example, the RMIT University AGR (illustrated above) has three linked SMART Boards and a motorised drop-down fourth screen set at 45° from the front wall: during an AG session presentation from the RMIT AGR, this fourth screen would be used for the videos of the remote audiences (so that the presenter can see the remote audiences). Many variations exist: for example, the AGR at The Mathematical Institute, University of Oxford, uses a front wall of three screens consisting of two inert projection screens and a SMART Board as the middle screen. One of the front screens is used as an audience screen which is copied to a fourth inert screen at the rear of the AGR.

Many presenters like to be able to work with two screens. One screen is used for the main presentation (pdf slides, say) and one screen for demonstrations (using software such as Maple, MATLAB, etc.) or for digital ink (handwritten asides, worked examples or sketches). The presenter can choose to enable, or not, remote control of the software being used: for example remote collaborators can take over the mouse and control the software (which might be Maple, Excel, Word, etc.). This facility is provided by the AG using VNC for the data/software stream.

AG for collaborative research and teaching

AGRs have been well established and widely used for collaborative research. For the last three years, about 90 regional and coast-to-coast seminars, in mathematics and computer science, have been conducted via a network of Canadian AGRs [1]. Recently Australia and the UK have led the use of AGRs for collaborative teaching of advanced courses in the mathematical sciences.

The International Centre of Excellence for Education in Mathematics (ICE-EM, the education arm of AMSI) is coordinating and has partially funded the introduction of a national network of AGRs located in mathematics departments at AMSI member universities. All of these AGRs run a Windows environment for applications. However the AG can also be run under UNIX or Mac environments.

In 2008, there are 11 AGRs operating in mathematics precincts, and there are about 30 additional AG nodes in Australia, some of which could be used by mathematical science departments. Most of the 38 universities in Australia have access to an AGR. In December 2007, an AMSI special one-day seminar was hosted in the RMIT AGR with 16 remote AGRs participating.

The first three mathematical science departments with AGRs ran a pilot program in semester two of 2006: collaboratively teaching Honours mathematics and statistics courses. During 2008, all of the AMSI AGRs are engaged with this national collaborative teaching program, coordinated by the ICE-EM, offering 17 Honours maths and statistics courses (see the ICE-EM website [3]). Students, with the approval of their home university, can take courses for credit toward their Honours degree. All AMSI member universities are invited to participate in this national collaborative teaching program (as well as other AMSI AGR activities such as seminars and workshops).

Six centres in the UK, funded by the Engineering and Physical Sciences Research Council for five years, commenced in October 2007 the teaching of broadening courses for PhD students [2]. Two of these centres use AG technology. They are the MAGIC consortium of 15 universities (see http://www.maths.dept.shef.ac.uk/magic/index.php) and the Taught Course Centre which is a collaboration between the mathematics departments at the Universities of Bath, Bristol, Imperial, Oxford and Warwick (see http://tcc.maths.ox.ac.uk/). A lot of information is freely available from these websites. Many courses are run and the AGRs are timetabled with these teaching commitments for most of each weekday during term.

AGR issues for mathematicians

There are many variations to how AGRs are constructed and how they operate. Mathematicians are often highly computer literate, but are not computer scientists. It is necessary to have good support from IT and network specialists. However it's desirable for standard connections to operate essentially in a turnkey mode (to minimise the specialist IT support that's needed): limited success with turnkey operation has been reported, but this is still an important goal.

ICE-EM has provided AGR teaching guidelines and presentation guidelines (see [4], although these will be revised soon). These include a requirement for the lecturer to have training in AGR use. Training programs are under development: at RMIT the IT Training group, with advice from me, has developed and delivered lessons interactively in small groups. These lessons are not immediately transferable to other institutions for two reasons: the AGRS are different (RMIT is the only one to have linked multiple SMART Boards) and RMIT usage of the mathematical software Maple is atypical.

Each AGR lecturer must be highly organised with prepared teaching and assessment materials and needs to be aware of variable student backgrounds. For AGR courses there are additional requirements for the use of electronic materials and access to these by the students, and pastoral care of the remote students.

At the ICE-EM, we wish to provide advice and support for effective use of the local AGR and remote AGRs. We want lecturers to be able to choose from a wide range of pedagogical styles and software. Typically, lecture notes are prepared as pdf files. The lecturer also uses some mathematical software for demonstrations and/or digital ink. Students submit work done by hand and/or software. Files are exchanged by email or web course management packages such as BlackBoard.

We are currently preparing advice on digital ink and annotations for asides and highlighting during the lecture (laser pointers are not effective) and for marking student work (as a pdf file and marked using a TabletPC and PDF Annotator or Jarnal, say).

Conclusion

The Access Grid is a system that provides video-conferencing and many more features to enable very rich multi-nodal remote collaborations in research and teaching. The mathematics community, especially in Australia and the UK, is leading the way with collaborative teaching of advanced mathematics across networks of AGRs.

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Bill Blyth is Associate Professor (Adjunct) of computational mathematics at RMIT University and was Head of the Department of Mathematics for $6\frac{1}{2}$ years. He is Chair of the Engineering Mathematics Group of Australia, a Center Affiliate at the International Centre for Classroom Research (at the University of Melbourne), led the design, construction and initial delivery phases of the RMIT University AGR and is currently at The Australian Mathematical Sciences Institute, AMSI, as the national coordinator of AMSI's Access Grid Room project. His PhD was in theoretical physics at Imperial College, London. He has an unusually broad range of research interests in mathematics education (in technology-rich classrooms) and the numerical solution of differential and integral equations. He has published more than 60 refereed papers.

Communications

A graduate modelling workshop: Canadian style

Nev Fowkes* **and Phil Broadbridge****

The Pacific Institute for the Mathematical Sciences (PIMS) held the 11th Graduate Industrial Mathematics Modelling Camp (GIMMC) and the 12th Industrial Problem Solving Workshop (IPSW) from 9 June to 20 June 2008 in Regina, Saskatchewan, Canada.

A total of 30 (mainly) postgraduate students from throughout the world attended the GIMMC, including four students from Australia (Thu Giang Nguyen (University of South Australia), Asef Nazari (University of Ballarat), Melanie Roberts (University of Western Australia) and Roslyn Hickson (Australian Defence Force Academy, a college of UNSW)). Almost all of the students remained on for the IPSW held during the second week and they were joined by a few local academics. In essence the GIMMC was the entrée, and IPSW the main course; a very effective arrangement. Support for attendance was provided by PIMS, and AMSI also provided support for the Australian representatives. This is the second consecutive year for which AMSI has sponsored Australian students to attend the PIMS Canadian GIMMC.

At the graduate workshop students were invited to address the problems presented by five appointed mentors and they presented the results they obtained at the end of GIMMC. Additionally students presented detailed reports. The IPSW format was similar with the problems being presented by industry representatives (similar to our MISG). The application areas ranged across combinatorics, financial mathematics, statistics and continuum modelling. A few of the problems will be described here.

Photo: Mike Esprit

The first example was a local one. Regina has a large casino (Casino Regina) run by the Saskatchewan Gaming Corporation which is a major tourist attraction and which reputedly offers a better deal for gamblers than Las Vegas. Over the long term the casino does very well (thank you) as you might expect, but there can be bad days (for the casino) and even bad weeks, and also particular tables can record a

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bad run (for example a $12,000 loss over a day on a table). The total cash drop on a typical day is about $300,000 and the actual daily gross result is quite volatile; it's a nervous business. Les Cloutier keeps an eye on the numbers and it is his job to detect any 'irregularities' (possibly due to cheating, theft, operator or equipment problems) in the recorded take from the various tables and if necessary call for management intervention. Over the years he has developed an eye for such irregularities but he was looking for an objective backup 'tool' for assistance. Whilst this superficially seems to be a standard statistical and/or probabilistic task, sifting through and analysing available data and presenting results in an immediately available form was challenging.


Photo: Gary Cowles

Snow clearance is, of course, an important and expensive task for towns in Canada. Ideally one would like to have snowplows traverse roads just once, clearing snow as they go, but dead runs can't be avoided. However, by a judicious choice of paths for the available snowplows one can minimise the number of dead runs or total cost, and perhaps even save the purchase of a new snowplow. This was the GIMMC problem posed by one of the mentors Ed Doolittle. Additional complications such as traffic lights, one- and two-way streets, primary and secondary roads, add to the challenge. A number of competing methods were examined and tested using real data.

A problem that is ideally suited for illustrating the power of scaling ideas in continuum modelling came from The Concrete Institute (Wits) and arose out of a South African MISG. During the construction of concrete dams, large slabs of concrete are poured and the hydration heat released as the concrete sets gives temperature rises typically in excess of 50°K. The conductivity of concrete is small so that very little of the heat generated within the concrete can be expelled through the surface during the construction period; the conductivity time scale for typical concrete slabs without cooling is about 20 years. The resulting temperature build up can lead to thermal stress induced cracking and resultant structural weakening and also can cause delays in construction. To reduce the effect, cooled water is piped through the concrete to extract as much of the hydration heat from within the slab as feasible without compromising its strength. Later the pipes are filled with concrete. The aim was to determine an optimal water pipe network to remove the heat.

Of course detailing possible winding paths of pipes through the slab is probably both impossible and pointless, so a simplified geometric model is essential. Additionally the time and length scales associated with heat transfer in the concrete and water are very different so that it is necessary to correctly assess the size of the various thermal interaction terms to arrive at a suitable approximate equation set. The students examined a simplified model consisting of a single pipe carrying water encased in an insulated hydrating concrete sleeve, and used scaling arguments to obtain estimates for the appropriate pipe spacing and length of, and flux through, pipes required to reduce temperature rise within the concrete to a prescribed level. It was an impressive effort.

The following oil exploration problem presented to GIMMC students came from BHP. The resistivity of a rock layer under the ocean is greatly increased by the presence of hydrocarbons and electromagnetic techniques have been used to detect the presence of such layers. To do this, a long wire carrying a high-voltage alternating current is trailed behind a ship close to the seabed and instruments previously placed on the sea bottom are used to record the electromagnetic field generated. The task was to use simple mathematical models to obtain explicit results for the dependence of the changed response on the depth and thickness of the (horizontal) oil layer. As many will appreciate, this is a demanding task more appropriate for a PhD project (there being six electrical and magnetic components to determine in an awkward geometry), so students were first asked to address a much simpler but analogous 1D thermal detection problem: There is a thin horizontal strip of material under the ground with conductivity different to that of the surroundings, is it possible to determine the location and thickness of the strip by observing differences in the thermal response of the ground due to an oscillatory heat source on the surface? Complete analytic solutions were obtained for the thermal detection problem and simple explicit results for the temperature change on the surface due to the thin strip, and using these results students described an oil exploration strategy. Surprisingly a small group of three students elected to directly attack the EMW problem. They obtained explicit results for the change in electromagnetic field components across a thin oil bearing layer and also they set out a plan for addressing the main problem; very impressive!

In the IPSW there were four problems presented including the following one posed by Professor Jack Tuszynski from the Division of Oncology Cross Cancer Institute in Edmonton, Canada. The current medical treatments for cancer include surgery, radiation therapy, gene therapy and chemotherapy, and much improved outcomes for patients have been achieved when these therapies have been used in combination. The general issue posed was how best to combine such therapies. It took us about a day to get to grips with the biology and medicine, after which we decided to restrict our attention to the general question: How can one evaluate and best improve the outcome for a patient by changing the application frequency of a particular drug or by using a combination of drugs? The drugs used generally fall into two categories: those that target particular stages of the cell cycle and those that interfere with the cell's environment. Antiangiogenesis drugs fall into the second category. These drugs inhibit the growth of blood vessels near the tumour, depleting the nutrient supply while enhancing the flow of other therapy drugs. Of course chemotherapy drugs are poisons which destroy both tumour and normal cells and effect the total physiology of the body, so balanced against the benefits are the side effects. This problem attracted a very large group of participants which broke into five groups examining various aspects using mechanistic, optimisation and probabilistic models. All the groups produced very worthy contributions of either fundamental or immediate practical interest. Most notably one of the groups developed a general dynamical system/probabilistic model that provides a practical framework for evaluating and modifying different treatment regimes. The Cancer Institute plans to set up a program based on this work.

This was a remarkably successful meeting, both technically and socially. The events were fully funded and represented a great opportunity for excellent students from throughout the world to learn from (and with) other students, and then immediately go on to apply their accumulated knowledge to real problems. The atmosphere during the second week was truly electric. Never before have I (Neville Fowkes) personally worked with such an enthusiastic group, and the outcome was outstanding; after the first week the students hit the ground running.

The four Australian students contributed well and thoroughly enjoyed the experience. Australia and Canada are realising the benefits of hosting international students to attend AMSI, MASCOS (Centre of Excellence for Mathematical and Statistics of Complex Systems), MITACS (the Mathematics of Information Technology and Complex Systems) and PIMS events. The attendance of Canadian students has enriched our AMSI-MASCOS industry workshops. PIMS encourages us to send students not only to the GIMMC but also to their annual summer school, which next year focuses on probability theory and stochastic modelling. We expect to send a couple of students to attend each of these events and will send details to university departments.

Neville Fowkes is a mathematical modeller and works mainly on continuum problems arising out of industrial, scientific and biological areas. He has participated in more than 30 maths-in-industry study groups (MISGs) held in many different countries. The companies he has worked with include Uncle Toby's, BHP, Hardie, CRA, ICI, duPont, British Steel, BrewTech, Mouldflow, Age Developments, Unilever, LNEC, Petronas and SAB. As an industrial modeller he has been asked to help initiate (and facilitate) MISGs in Indonesia and South Africa, and he has also helped set up industrial mathematics graduate and undergraduate programs in Portugal, China, Indonesia, Laos and Thailand. Neville has written a text on Mathematical Modelling based largely on problems arising out of his industrial mathematics experiences. This book has been used in Australia, UK and Europe (and possibly elsewhere) for industrial mathematics courses.

Ninth Australasian Conference on Mathematics and Computers in Sport

Neville de Mestre*

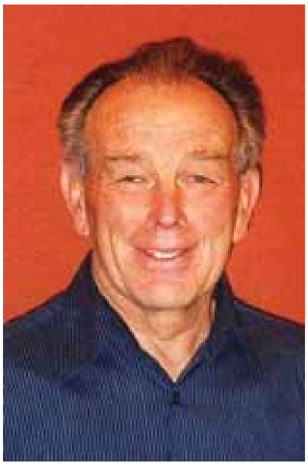
The ninth Australasian Conference on Mathematics and Computers in Sport was held at Twin Towns Resort, Tweed Heads from Monday 1 September to Wednesday 3 September. There were 34 participants from Australia, New Zealand and Europe. The principal speakers were Arnold Baca from the University of Vienna who spoke on 'Tracking Motion in Sport', and Ian Renshaw from Queensland University of Technology who spoke on 'Performance and Learning of Motor Skills'.

The conference was opened by Geoff Pollard, president of Tennis Australia, who also delivered three papers on tennis and golf. Geoff is currently undertaking a PhD at Swinburne University. There were 34 papers presented on topics that included cricket, tennis, badminton, Australian Rules football, rugby union, table tennis, volleyball, baseball, kayaking, sailing, bodysurfing and athletics.

The extra-curricula activity for this conference was held at the Tweed Heads Indoor Bowling Centre on Tuesday evening attended by 15 of the delegates.

The conference was directed by Neville de Mestre, while John Hammond from Lincoln University (UK) edited the refereed *Proceedings*. Copies of these are available from the MathSport group at $40 plus postage (email margnev@omcs.com.au).

The tenth conference will be held in Melbourne in 2010 with Anthony Bedford from RMIT as director.



Neville de Mestre retired from Bond University in 2004 as Emeritus Professor of Mathematics. Neville lectured at RMC Duntroon and ADFA from 1962 to 1989. He moved to Bond University as soon as it opened in 1989. In 1992 he inaugurated the Biennial Australasian Mathematics and Computers in Sport conferences, which eventually became the Mathsport Special Interest Group of ANZIAM. Neville has directed or co-directed seven of the first nine conferences. Neville has also been Chair of ANZIAM (during ICIAM 2003 in Sydney) and Deputy Chair on a number of occassions. He was director of ANZIAM conferences at Smiggin Holes (1971), Merimbula (1984) and Coolangatta (1998). His research interests include fluid mechanics, bushfires, sport and mathematical education at all levels. He founded the ACT Mathematics Centre which now travels Australia through Questacon. He recently gave the tenth G.S. Watson Memorial Lecture at La Trobe University (Bendigo).

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Higher Degrees and Honours Bachelor Degrees in Mathematics and Statistics completed in Australia in 2007

Peter Johnston*

This report presents data relating to students who completed Honours or Higher Degrees in Mathematics during 2007. The data is part of an on going project for the Australian Mathematical Society and should be read in conjunction with previous reports [1], [2], [3], [4], [5], [6], [7], [8] covering the period 1993–2006.

Appendix 1 presents data for students completing Honours degrees in 2007, at all universities in Australia. Within each institution, the data are broken down into male and female students and into the three traditional areas of mathematics: pure; applied and statistics. There is also the general category 'Mathematics' for institutions which do not differentiate between the conventional areas. Finally, there is an 'Other' category for newer areas of mathematics such as financial mathematics. Each category is further broken down into grades of Honours awarded. The table shows that in 2007 there were 167 Honours completions in Australia, with 113 (68%) receiving First Class Honours (compared with 106 out of 154 (69%) in 2006 and 105 out of 152 (69%) in 2005). It is pleasing to see a generally increasing trend in the number of Honours completions over the last 10 years.

Figure 1 presents the total number of students completing Honours degrees in Mathematics over the period 1959–2007. It shows that in 2007 the number of graduates rose above the levels of the period 2001–2003. The figure also shows the numbers of male and female students who completed Honours over the same time period. For last year the number of male students remained constant at 115, whereas there was an increase in the number of female students (52, up from 39).

Appendix 2 presents the data for Higher Degree completions in 2007. The data are broken down into Coursework Masters, Research Masters and PhD degrees, with the latter two divided into the three typical areas of mathematics. These data are also represented in Figure 2, as part of the overall Higher Degree data for the period 1959–2007, contrasting the most recent data with previous years.

(1) There was a record number of PhD completions. In 2007, there were 77 PhD completions, of which 56 were by male students and 21 by female students, one more than the previous record of 76 in 1995.
(2) The number of Research Masters completions has decreased after a spike last year.
(3) The number of Coursework Masters completions has rebounded to levels of 2005.

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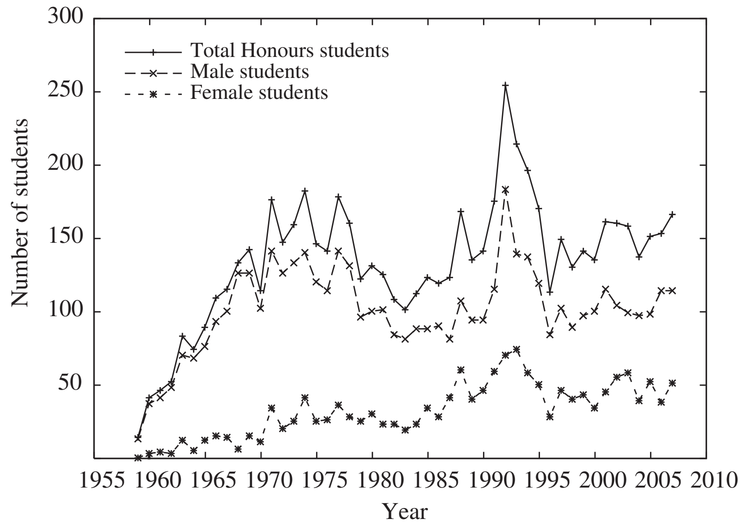

Figure 1. Number of Honours degrees completed in mathematics and statistics, 1959–2007.

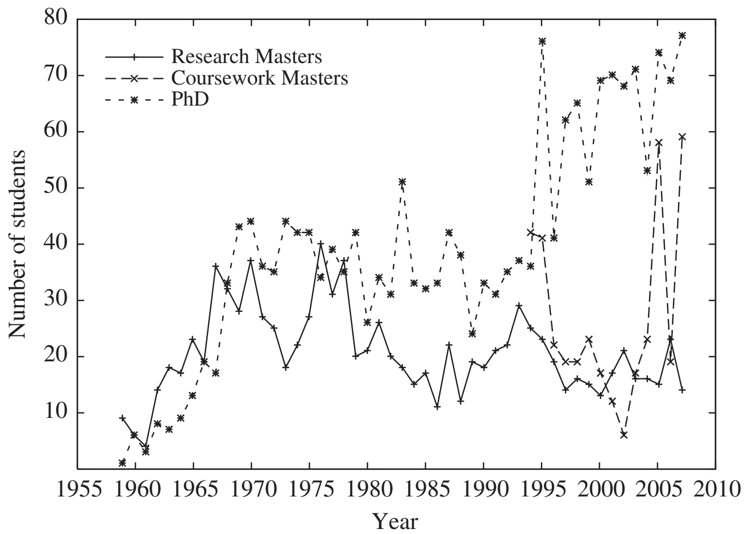

Figure 2. Number of research higher degrees completed in mathematics and statistics, 1959–2007.

For those who are interested in the finer details, the raw data is available from links on the web page http://www.cit.gu.edu.au/maths. There is an Excel spreadsheet containing the complete data for 2007 as well as spreadsheets containing cumulative data from 1959 for Honours, Research Masters and PhD degrees.

I would like to thank the many people who took the time and effort to collect this data and forward it to me. This year I received 30 out of a possible 38 responses to requests for data, similar to last year's response rate. Finally, if having read this report, you would like to contribute missing data for 2007, I can add it to the data on the website.

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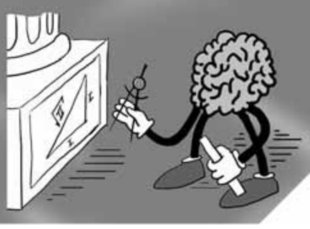
Appendix 1. Number of Honours degrees completed in mathematics and statistics, 2007.

Uni.	Sex	Maths				Pure				Applied				Statistics				Other				Honours
		I	IIA	IIB	III	I	IIA	IIB	III	I	IIA	IIB	III	I	IIA	IIB	III	I	IIA	IIB	III	Total
ACU	M																					0
	F																					0
ADF	M																					0
	F																					0
ANU	M	5																				5
	F	2																				2
BOU	M																					0
	F																					0
CDU	M																					0
	F																					0
CQU																						0
																						0
CSU	M																					0
	F																					0
CUT	M																					0
	F																					0
DKU	M									1												1
	F																					0
ECU	M													1								1
	F																					0
FDU	M																					0
	F																					0
GFU	M																					0
	F																					0
JCU																						0
																						0
LTU																						0
																						0
MDU	M						1															1
	F										1				1							2
MNU	M					4				1	1				1							7
	F					3	1			1	1			1	1							8
MQU	M	1																				1
	F																					0
QUT	M									4	2			1								7
	F									3	1			3								7
RMT	M							1		3					1							5
	F									2	1				1							4
SCU																						0
																						0
SUT	M																					0
	F																					0
UAD	M	1				2	2							2	2							9
	F					2																2
UBR																						0
																						0
UCB	M																					0
	F																					0
UMB	M					5				3	1		1	3	2							15
	F								1					1								2
UNC																						0
																						0
UNE	M						1															1
	F						1															1
UNS	M					5				2				3	2							12
	F					1				1	1											3
UQL	M					2				5												7
	F							1						1								2
USA	M									2												2
	F									1												1
USN	M					5	2			5	2	2		3								19
	F					1	1			2			1	3	1							9
USQ	M													1								1
	F																					0
UTM																						0
																						0
UTS	M	1													2		1					4
	F		1												1							2
UWA	M					1	1															2
	F					1						1			1							3
UWG	M					1				4	1			2	1			1	2	2		14
	F													1				3				4
UWS	M						1															1
	F																					0
VUT																						0
																						0
Totals		10	1	0	0	33	11	2	1	40	12	3	2	26	17	0	1	4	2	2	0	167

Appendix 2. Number of research higher degrees completed in mathematics and statistics, 2007

Uni.	Sex	Coursework Masters	Research Masters Pure	Applied	Statistics	Research Masters Total	PhD Pure	Applied	Statistics	PhD Total
ACU	M					0				0
	F					0				0
ADF	M					0		1	1	2
	F					0			1	1
ANU	M					0		4		4
	F			1		1				0
BOU	M					0				0
	F					0				0
CDU	M					0				0
	F					0				0
CQU						0				0
						0				0
CSU	M					0				0
	F					0				0
CUT	M	6				0		3	1	4
	F					0				0
DKU	M					0				0
	F					0				0
ECU	M					0				0
	F					0				0
FDU	M					0	1			1
	F					0				0
GFU	M					0				0
	F					0				0
JCU						0				0
						0				0
LTU						0				0
						0				0
MDU	M					0	1			1
	F					0		1	1	2
MNU	M					0		7	1	8
	F					0		1		1
MQU	M					0	1			1
	F					0				0
QUT	M	1			1	1		1		1
	F	1				0		1	1	2
RMT	M	14				0		4		4
	F	6				0				0
SCU						0				0
						0				0
SUT	M					0				0
	F					0				0
UAD	M		2	3		5		2	1	3
	F					0				0
UBR						0				0
						0				0
UCB	M					0				0
	F					0				0
UMB	M					0	1	3		4
	F			1	1	2		3		3
UNC						0				0
						0				0
UNE	M					0				0
	F					0				0
UNS	M	10	1			1	3	1		4
	F	2				0				0
UQL	M	7				0	1	5	1	7
	F	3		1		1	2			2
USA	M					0		1		1
	F					0		3		3
USN	M					0				0
	F			1		1	4	2	1	7
USQ	M					0	2			2
	F	1				0				0
UTM						0				0
						0				0
UTS	M					0		1		1
	F					0				0
UWA	M					0	3	2	1	6
	F					0				0
UWG	M	5	1	1		2		1	1	2
	F	3				0				0
UWS	M					0				0
	F					0				0
VUT						0				0
						0				0
Totals		59	4	8	2	14	19	47	11	77



Technical papers

Osculation by circumcircles of a pantograph

John Boris Miller*

Abstract

The circumcircles of the cyclic quadrilaterals in the family $\mathrm{Q}(\Lambda)$ of all quadrilaterals for which Λ is the median parallelogram osculate an octic curve, which is described. When Λ is a rhombus this curve reduces to a pair of double ellipses.

In a previous paper [1] we showed that, given a parallelogram $\Lambda = \mathrm{FGHI}$, there exists a doubly infinite family $\mathcal{Q}(\Lambda)$ of quadilaterals, namely all quadrilaterals having three (and therefore all four) of the vertices of Λ as the midpoints of its sides, taken in order; and within $\mathcal{Q}(\Lambda)$ there exists a singly infinite family of cyclic quadrilaterals, whose centres have as locus a rectangular hyperbola $\mathrm{Cen}(\Lambda)$ through the vertices of Λ (or an orthogonal line-pair, namely the diagonal lines, when Λ is a rhombus). We now wish to describe the curve Γ which these circumcircles osculate. Let $\mathcal{C}(\Lambda)$ denote the family of circumcircles.

Γ for a rhombus

We start with the easier case, when Λ is a rhombus. Write f for the side length, ω for the angle, take $0 < \omega \leq \frac{1}{2}\pi$ and $s = \sin \frac{1}{2}\omega$, $c = \cos \frac{1}{2}\omega$, O for the centre of Λ, and take cartesian axes Ox, Oy with Ox parallel to and in the direction FG[1]. The diagonal lines are $\mathrm{s}x - \mathrm{c}y = 0$, $\mathrm{c}x + \mathrm{s}y = 0$. Let $\Theta(u)$ denote the circle of the family $\mathcal{C}(\Lambda)$ having centre $\mathrm{U}(u) = (\mathrm{c}u, \mathrm{s}u)$, $u = |\mathrm{OU}|$, which lies on the longer diagonal IG. See Figure 1.

The equation of $\Theta(u)$ is $\theta(u) = 0$ where (see [1, Theorem 5])

$$\theta(u) := x^2 + y^2 - 2(\mathrm{c}x + \mathrm{s}y)u - f^2 - \mathrm{c}^2\mathrm{s}^{-2}u^2 \,. \tag{1}$$

To find the osculated curve we look for the limiting positions of the intersections of circles $\Theta(u)$, $\Theta(u_1)$ as $u \to u_1$. From the equations of these two circles we find

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[1] The notation of [1] is here amended as follows: symbols S, C, E, X, ξ, η are replaced by s, c, e, O, x, y respectively.

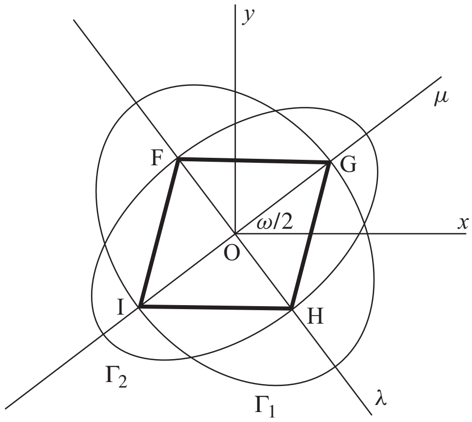

Figure 1. Rhombus Λ and the two ellipses Γ_1, Γ_2.

by subtraction

$$\mathrm{c}x + \mathrm{s}y = -\mathrm{c}^2(2s)^{-1}(u + u_1)\,; \tag{2}$$

then eliminating x from this and $\theta(u_1) = 0$ gives

$$\mathrm{s}^4y^2 + \mathrm{s}^3\mathrm{c}^3(u + u_1)y + 4^{-1}\mathrm{c}^4(u^2 + u_1^2 + 2(1 + 2\mathrm{s}^2)uu_1) - \mathrm{s}^4\mathrm{c}^2f^2 = 0\,.$$

The discriminant of this quadratic in y is

$$D := -\mathrm{s}^4\mathrm{c}^2(4^{-1}\mathrm{c}^4(u + u_1)^2 + \mathrm{s}^2\mathrm{c}^2uu_1 - \mathrm{s}^4f^2)\,,$$

and from this we can conclude the following:

Lemma 1. (i) *If* $|u_1| \geq 2\mathrm{s}^2\mathrm{c}^{-2}f$ *then* $\Theta(u_1)$ *does not intersect* $\Theta(u)$ *for any* $u(\neq u_1)$ *of the same sign as* u_1*: one circle contains the other.*

(ii) *If* $|u_1| \leq \mathrm{s}^2\mathrm{c}^{-1}f$ *then* $\Theta(u_1)$ *intersects all circles* $\Theta(u)$ *for which* $|u - u_1| < \varepsilon$*, for some* ε *depending upon* u_1*.*

Thus in case (ii) we can expect osculation, and only in this case. Let $u \to u_1$ in (1), (2) to obtain the coordinates (x, y) of the two limiting positions of the points of intersection of the two circles. (In effect, the calculation has been to eliminate u from the equations $\theta(u) = 0$, $\theta'(u) = 0$.) Make a clockwise rotation through angle $\frac{1}{2}(\pi - \omega)$ from axes Oxy to new axes O$\lambda\mu$: there is an unexpected disappearance of the u_1 terms from the algebra, and we arrive at

$$\Gamma_1\colon \lambda^2 + \mu^2\mathrm{c}^{-2} = f^2\,.$$

This is the equation of an ellipse with eccentricity s, foci at F, H, and passing through G, I; it is the curve osculated by the circumcircles $\Theta(u)$ when $|u| \leq \mathrm{s}^2\mathrm{c}^{-1}f$. The points on the diagonal through I, G where $u = \pm\mathrm{s}^2\mathrm{c}^{-1}f$ lie between G and I. As the centre U(u) of $\Theta(u)$ moves from $u = +\mathrm{s}^2\mathrm{c}^{-1}f$ to $u = -\mathrm{s}^2\mathrm{c}^{-1}f$, the points of osculation (there are two of them) move from coincidence at I, around the ellipse in opposite directions to G. Since each circle osculates at two points, we deem this ellipse to be double.

All this calculation assumes that U is on the line $\lambda = 0$ containing IG; but of course there are similar outcomes for the line $\mu = 0$ along the shorter diagonal FH. Here $\mathrm{U}(u)$ has coordinates $x = -\mathrm{s}u$, $y = \mathrm{c}u$, and the osculating curve for the circles with centres $\mathrm{U}(u)$ on this line is found to be the ellipse

$$\Gamma_2 \colon \lambda^2 \mathrm{s}^{-2} + \mu^2 = f^2 \,,$$

having eccentricity c, foci at G, I, and passing through F, H. To summarize:

Theorem 1. *When Λ is a rhombus with sidelength f and angle ω, $0 < \omega \le \frac{1}{2}\pi$, the circumcircles $\Theta(u)$ of the cyclic quadrilaterals in $\mathcal{Q}(\Lambda)$ osculate an octic curve, consisting of the two ellipses Γ_1 and Γ_2, each counted twice. The osculation occurs for $|u| \le \mathrm{s}^2\mathrm{c}^{-1} f$, $|u| \le \mathrm{c}^2\mathrm{s}^{-1} f$ respectively. The ellipses have major semi-axes of equal lengths f and minor semi-axes of lengths $\mathrm{c}f$, $\mathrm{s}f$ respectively.*

Γ for a non-rhombus

Now suppose that Λ is a non-rhomboidal parallelogram. The problem is of an altogether greater level of difficulty. Let the sidelengths be f, g with $f > g$, the angle be ω, with $0 < \omega \le \frac{1}{2}\pi$. We take the same pairs of axes $\mathrm{O}x, \mathrm{O}y$ and $\mathrm{O}\lambda, \mathrm{O}\mu$ as before, so that $\lambda = \mathrm{s}x - \mathrm{c}y$, $\mu = \mathrm{c}x + \mathrm{s}y$, which reduces the equation of the locus of circumcentres $\mathrm{Cen}(\Lambda)$, a rectangular hyperbola, to

$$\lambda\mu = \mathrm{e}^2 \,, \quad \text{where } \mathrm{e} := \frac{\sqrt{(f^2 - g^2)\mathrm{sc}}}{2} \,. \tag{3}$$

See [1, Equation (3)]. We change to a parameter t (differently defined from u): the circumcircle $\Theta(t)$ with centre $\mathrm{T} = (\mathrm{e}t, \mathrm{e}t^{-1})$ on the hyperbola has equation $\psi(t) = 0$ where

$$\psi(t) := \lambda^2 + \mu^2 - 2e(\lambda t + \mu t^{-1}) - \mathrm{e}^2(\mathrm{s}^2\mathrm{c}^{-2}t^2 + \mathrm{c}^2\mathrm{s}^{-2}t^{-2}) - h^2 \,, \quad h^2 := \tfrac{1}{2}(f^2 + g^2) \,. \tag{4}$$

(See [1, Theorem 5].) To find the curve of osculation Γ we have to eliminate t from the two equations $\psi(t) = 0$, $\psi'(t) = 0$. This involves equating to zero their resultant, an 8×8 determinant (that is, the discriminant of ψ; see [6, pp. 83–88]). Such a determinant has 40 320 terms, but mercifully most of these are zero. The outcome, after heavy algebra (or more simply by use of Mathematica), is:

$$\begin{aligned} \mathrm{R} = {} & \mathrm{s}^2\mathrm{c}^2\zeta^8 + (\mathrm{c}^4\lambda^2 + \mathrm{s}^4\mu^2)\zeta^6 + \mathrm{s}^2\mathrm{c}^2(\lambda^2\mu^2 - 20\lambda\mu\mathrm{e}^2 - 8\mathrm{e}^4)\zeta^4 \\ & - 18\mathrm{e}^2[\mathrm{c}^4\lambda^3\mu + \mathrm{s}^4\lambda\mu^3 + 2\mathrm{e}^2(\mathrm{c}^4\lambda^2 + \mathrm{s}^4\mu^2)]\zeta^2 - 27\mathrm{e}^4\mathrm{c}^6\mathrm{s}^{-2}\lambda^4 \\ & - 27\mathrm{e}^4\mathrm{s}^6\mathrm{c}^{-2}\mu^4 + 16\mathrm{e}^8\mathrm{s}^2\mathrm{c}^2 - \mathrm{e}^2\mathrm{s}^2\mathrm{c}^2[16\lambda^3\mu^3 + 6\mathrm{e}^2\lambda^2\mu^2 + 48\mathrm{e}^4\lambda\mu] \\ = {} & 0 \,, \end{aligned}$$

where ζ^2 is written for $\lambda^2 + \mu^2 - \frac{1}{2}(f^2 + g^2)$, and an irrelevant constant factor has been omitted. Simplifying further by substituting for ζ^2 we eventually arrive at an octic equation in λ and μ, and the following result:

Theorem 2. *When Λ is not a rhombus, the curve of osculation of the family $\mathcal{C}(\Lambda)$ of circumcircles of the pantograph $\mathcal{Q}(\Lambda)$ referred to axes $\mathrm{O}\lambda\mu$ has equation $\mathrm{R} = 0$, where*

$$\mathrm{R} := \mathrm{R}_8 - \mathrm{R}_6 + \mathrm{R}_4 + \mathrm{R}_2 + \mathrm{R}_0 \,, \tag{5}$$

each R_j *denoting a homogeneous polynomial of degree* j *in* λ *and* μ*, as follows:*

$$
\begin{aligned}
\mathrm{R}_8 &= \mathrm{c}^2\lambda^8 + (1+3\mathrm{c}^2-\mathrm{c}^4)\lambda^6\mu^2 + (3+2\mathrm{s}^2\mathrm{c}^2)\lambda^4\mu^4 + (1+3\mathrm{s}^2-\mathrm{s}^4)\lambda^2\mu^6 + \mathrm{s}^2\mu^8\,,\\
\mathrm{R}_6 &= h^2\mathrm{c}^2(\mathrm{s}^2+3)\lambda^6 + \mathrm{e}^2\mathrm{c}^2(20\mathrm{s}^2+18\mathrm{c}^2)\lambda^5\mu + h^2(3+8\mathrm{c}^2-5\mathrm{c}^4)\lambda^4\mu^2\\
&\quad + \mathrm{e}^2(20\mathrm{s}^2\mathrm{c}^2+18)\lambda^3\mu^3 + h^2(3+8\mathrm{s}^2-5\mathrm{s}^4)\lambda^2\mu^4\\
&\quad + \mathrm{e}^2\mathrm{s}^2(20\mathrm{c}^2+18\mathrm{s}^2)\lambda\mu^5 + h^2\mathrm{s}^2(\mathrm{c}^2+3)\mu^6\,,\\
\mathrm{R}_4 &= [3h^4(1-\mathrm{s}^4) + \mathrm{e}^4\mathrm{s}^{-2}\mathrm{c}^2(-27+18\mathrm{s}^2+\mathrm{s}^4)]\lambda^4 + h^2\mathrm{e}^2\mathrm{c}^2(40-22\mathrm{c}^2)\lambda^3\mu\\
&\quad + [3h^4\mathrm{s}^2\mathrm{c}^2(1+3\mathrm{s}^2\mathrm{c}^2) + \mathrm{e}^4(50\mathrm{s}^2\mathrm{c}^2-36)]\lambda^2\mu^2 + h^2\mathrm{e}^2\mathrm{s}^2(40-22\mathrm{s}^2)\lambda\mu^3\\
&\quad + [3h^4(1-\mathrm{c}^4) + \mathrm{e}^4\mathrm{s}^2\mathrm{c}^{-2}(-27+18\mathrm{c}^2+\mathrm{c}^4)]\mu^4\,,\\
\mathrm{R}_2 &= [-h^6\mathrm{c}^2(1+3\mathrm{s}^2) + 4h^2\mathrm{e}^4\mathrm{s}^2\mathrm{c}^4(9-5\mathrm{s}^2)]\lambda^2 - 4\mathrm{s}^2\mathrm{c}^2(5\mathrm{e}^2h^4+12e^6)\lambda\mu\\
&\quad + [-h^6\mathrm{s}^2(1+3\mathrm{c}^2) + 4h^2\mathrm{e}^4\mathrm{s}^4\mathrm{c}^2(9-5\mathrm{c}^2)]\mu^2\,,\\
\mathrm{R}_0 &= \mathrm{s}^2\mathrm{c}^2(h^4-4\mathrm{e}^4)^2.
\end{aligned}
$$

Here e, h *are as defined in* (3), (4) *and* $\mathrm{s} = \sin\frac{1}{2}\omega$*,* $\mathrm{c} = \cos\frac{1}{2}\omega$*.*

The expression R is homogeneous of degree 8 in λ, μ, h and e, with 21 terms in various powers of λ and μ. Evidently R possesses certain symmetries. For example, $\mathrm{R}(\lambda,\mu) = \mathrm{R}(-\lambda,-\mu)$, and (if we show ω as a variable in R) $\mathrm{R}(\lambda,\mu,\omega) = \mathrm{R}(\mu,\lambda,\pi/2-\omega)$.

To find the point where, for a given t, the circle $\Theta(t)$ touches the osculating curve, we have to solve the equations $\psi(t)=0$, $\psi'(t)=0$ for λ,μ. This involves solving a simultaneous pair of equations of the form $\mathrm{X}^2+\mathrm{Y}^2=\mathrm{R}^2$, $\mathrm{AX}+\mathrm{BY}=\mathrm{K}$, where X,Y are linear forms in λ and μ. Doing this, we find that the solution is real if and only if the expression

$$\Delta(t) := -t^4\mathrm{e}^2\mathrm{s}^2\mathrm{c}^{-4} + t^2h^2 + 3\mathrm{e}^2\mathrm{s}^{-2}\mathrm{c}^{-2} + t^{-2}h^2 - t^{-4}\mathrm{e}^2\mathrm{c}^2\mathrm{s}^{-4}$$

is positive or zero. This expression has the same sign as $\mathrm{s}^4\mathrm{c}^4t^4\Delta(t)$; so write $v=t^2$ and consider the quartic polynomial

$$\mathrm{D}(v) := -v^4\mathrm{e}^2\mathrm{s}^6 + v^3\mathrm{s}^4\mathrm{c}^4h^2 + 3v^2\mathrm{e}^2\mathrm{s}^2\mathrm{c}^2 + v\mathrm{s}^4\mathrm{c}^4h^2 - \mathrm{e}^2\mathrm{c}^6\,.$$

By examining the properties of D and its derivatives we find that the graph of $w=\mathrm{D}(v)$ cuts the w-axis at $w=-\mathrm{e}^2\mathrm{c}^6$, that $w\to-\infty$ as $v\to\infty$, and there is a single maximum of D on $v>0$. Moreover $\mathrm{D}(v)>0$ when $v=(\mathrm{cs}^{-1})^{3/2}$. Thus D has exactly two positive zeros, at say $v=\alpha^2,\beta^2$, between which it is positive, and therefore the simultaneous equations $\psi(t)=0$, $\psi'(t)=0$ have a real solution for λ, μ when $\alpha<|t|<\beta$. We conclude:

Lemma 2. *The circles* $\Theta(t)$ *osculate the octic curve* (5) *when* $-\beta\le t\le-\alpha$ *and when* $\alpha\le t\le\beta$*, but for no other values of* t*. Here* α,β *are two numbers satisfying*

$$0<\alpha<(\mathrm{cs}^{-1})^{3/4}<\beta\,.$$

The formulae for α and β are complicated.

The nature of the octic curve Γ

We can discover the shape of Γ by the use of *Mathematica*. The following program gives a plot of Γ and its originating parallelogram Λ, for arbitrary values of f, g and ω, where as usual we assume $0 < g < f$ and $0 < \omega < \pi/2$.

```
p1 = e^2 c^4 + 2 e s^2 c^2 μ t - s^2 c^2 (λ^2 + μ^2 - h^2) t^2
     + 2 e s^2 c^2 λ t^3  + e^2 s^4 t^4;
q1 = - e^2 c^4 - e s^2 c^2 μ t + e s^2 c^2 λ t^3 + e^2 s^4 t^4;
p2 = Expand[p1/.{λ -> s x - c y, μ -> c x + s y,
     e -> Sqrt[(f^2 - g^2) s c]/2, h -> Sqrt[(f^2 + g^2)/2]}];
q2 = Expand[q1/.{λ -> s x - c y, m -> c x + s y,
     e -> Sqrt[(f^2 - g^2) s c]/2, h -> Sqrt[(f^2 + g^2)/2]}];
r1 = Resultant[p2, q2, t];
r2 = r1/.{s -> Sin[ω/2], c -> Cos[ω/2]};
r3 = FullSimplify[r2];
r4 = r3*(-2199023255552/((f^2 - g^2)^4 Sin[ω]^18));
Manipulate[Show[ContourPlot[r4, {x, -30, 30}, {y,- 30,30}, Axes -> True,
   AxesLabel -> {x,y}, Frame -> False, Contours -> {0},
   ContourStyle -> {Thick, Black}], ListPlot[{{(-f + g Cos[ω])/2,
   g Sin[ω]/2},{(f + g Cos[ω])/2, g Sin[ω]/2},{(f - g Cos[ω])/2,
   - g Sin[ω]/2},{(- f - g Cos[ω])/2,- g Sin[ω]/2},{(-f + g Cos[ω])/2,
   g Sin[ω]/2}}, PlotStyle -> {Thickness[0.005], Black},
   PlotJoined -> True]], {ω, 0., Pi/2}, {f,0,20}, {g,0,f}]
```

Here p1, q1 are the functions $\psi(t)$, $\psi'(t)$ respectively, shorn of irrelevant factors; p2, q2 express these functions in terms of x and y, and substitute for e and h. Then r1 is the resultant, eliminating parameter t; this has 1326 summands. Expression r2 substitutes the trigonometric values of s and c; r3 finds a constant factor, allowing this to be divided out for r4.

The equation $\mathrm{R} = 0$ defining Γ referred to axes Oxy is then essentially the equation $\mathrm{r4} = 0$. To plot the implicit equation $\mathrm{R(x, y)} = 0$, we ask for the contour $z = 0$ of the surface $z = \mathrm{R}(x, y)$. It may be necessary to substitute for some labels such as r1 their full expressions. ListPlot[...] gives the parallelogram Λ. The two graphs can be manipulated together using sliders for ω, f and g. Fixing f, we get a two-parameter family of octics, depending on parameters f/g and ω from the dimensions for the parallelogram.

Figure 2 shows Γ for three choices of the shape of Λ. When g is sufficiently less than f(first plot), Γ takes the form of two identical but rotated smooth egg-shaped circuits, overlapping, so that a transversal line cuts the curve in at most 4 real points, a general pencil of real lines through an external point contains 4 tangents, there are 2 crunodes, no cusps, 2 double tangents, and no inflexions. As g nears f the curve becomes more erratic (middle plot), until the curve seemingly approximates to a four-bean quartic[2] having 4 circuits and 28 real bitangents, and thus approximates to the pair of ellipses which we met aready for the case when $g = f$ and Λ is a rhombus. Γ always passes through the vertices of Λ.

[2]Presumably the quartic is doubled. For four-bean quartics see [2, Chapter XIX, p. 342].

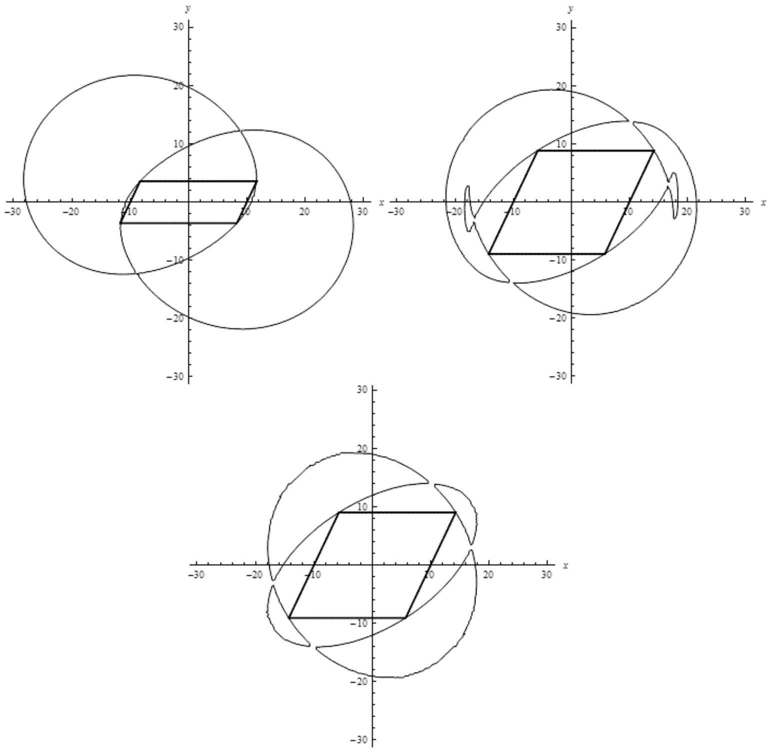

Figure 2. Three depictions of Γ, for $f = 20$ and $\omega = 1.28$ radians. They are the cases $g = 8$, $g = 19.68$ and g slightly less than 20, respectively.

The process of osculation can be observed by a second Mathematica program. See Figure 3. The interval $[\alpha, \beta]$ of Lemma 2 in the stated case is approximately $[0.5, 6]$. As t increases from 0.5, the osculation point starts somewhere about $x = -6$, $y = -20$, it splits immediately into two points which move in opposite directions around the righthand circuit of Γ until they meet again when $\mathrm{t} \approx 6$; thereafter $\Theta(\mathrm{t})$ and Γ lose contact.

The values for the Plücker numbers of Γ implied by the previous paragraph do not satisfy Plücker's equations[3]. We must conclude either that the equation of Γ contains a squared factor, so that part or all of Γ is described twice, or that there are more points on the curve than are shown in the diagrams. A study of the Mathematica plots fails to reveal any extra real parts to the curve. Separately, these plots suggest that R factors in the form $\mathrm{R}(x, y) = \mathrm{M}(x, y).\mathrm{M}(-x, -y)$, where M would of course be a quartic; but Mathematica finds no factors other than constants, even over $\mathbb{C}[x, y]$. So there remain some unresolved questions about Γ.

The rectangular hyperbola Cen(Λ) is the *evolute* of Γ, being the locus of its centre of curvature. It is not asserted that all points of Γ are points of osculation; though

[3]For a discussion of Plücker numbers and equations see [2, Chapter VIII].

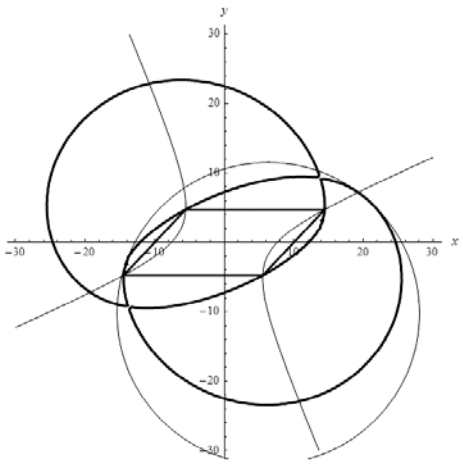

Figure 3. Osculation, the particular case when $f = 20$, $g = 13$, $\omega = 0.815$, showing the circle $\Theta(t)$ for $t = 2.6$.

this is evidently the case when g is much less than f, from the Mathematica plots. The circles are bitangent circles to Γ; the minimum of their radii is f.

Acknowledgements

I received advice on the use of Mathematica from Samuel Blake at an intermediate stage of the investigation, and valuable help with Mathematica from Chetiya Sahabanda of Technical Support, Wolfram Research, Inc. I thank also the School of Mathematical Sciences of Monash University for issuing me with a site licence for Mathematica. Figure 1 was drawn using TurboCAD [5], and is not to scale. Figures 2 and 3 were drawn in Mathematica [3].

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Minimal faithful permutation degrees of finite groups

Neil Saunders*

Abstract

We calculate the minimal degree for a class of finite complex reflection groups $G(p,p,q)$, for p and q primes and establish relationships between minimal degrees when these groups are taken in a direct product.

Introduction

The minimal faithful permutation degree $\mu(G)$ of a finite group G is the least non-negative integer n such that G embeds in the symmetric group $Sym(n)$. That is, $\mu(G)$ is the degree of the smallest faithful permutation representation of G, where a permutation representation is a group homomorphism from G to $Sym(X)$ for some set X.

It is well known that when a group G acts on a finite set X, the G-orbits induce an equivalence relation on X and we can write

$$X = X_1 \sqcup \ldots \sqcup X_r,$$

where each X_i represents a G-orbit.

The restriction of G to one of its orbits on X_i is *transitive* and we can easily verify that this action is equivalent to a right action on a set of cosets G/G_i where G_i is the stabiliser of a point in X_i.

Specifically, fix a point $x_i \in X_i$ and define a map θ from X_i to G/G_i by $\theta(x) = G_i h$ where $h \in G$ and $x_i h = x$. It is easy to see that Figure 1 commutes for all $x \in X_i$ and $g \in G$.

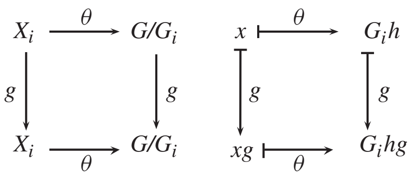

Figure 1.

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The kernel of the action of G on G/G_i is the subgroup $\bigcap_{g\in G} g^{-1}G_i g$ called the *core* of G_i, denoted by $\mathrm{core}(G_i)$. This is the largest normal subgroup of G that is contained in G_i. We call G_i *core-free* if $\mathrm{core}(G_i)$ is trivial.

Given that each permutation representation is a disjoint union of transitive representations which are equivalent to right actions on a set of cosets, we may abbreviate the information defining this permutation representation of G on X by the list $\{G_1, \ldots, G_r\}$ where each G_i represents the stabliser of a point in the orbit X_i. We will often denote such a collection of subgroups by $\mathcal{R}$ and refer to it as the representation of G. The elements of $\mathcal{R}$ are called *transitive constituents* and if $\mathcal{R}$ consists of just one subgroup G_0 say, then we say that $\mathcal{R}$ is transitive, in which case G_0 is core-free by faithfulness.

We may now restate the definition of the minimal faithful permutation degree of a finite group G.

$\mu(G)$ is the smallest value of $\sum_{i=1}^{n} |G\colon G_i|$ for a collection of subgroups $\mathcal{R} = \{G_1, \ldots, G_n\}$ satisfying $\bigcap_{i=1}^{n} \mathrm{core}(G_i) = \{1\}$

Thus the problem of finding the minimal permutation degree of a finite group presents a dichotomy relating to its lattice of subgroups. On the one hand we want to include as many subgroups in the collection so we can satisfy the condition that the intersection of their cores has to be trivial. On the other hand, we would like this collection to be as small as possible and the subgroups to be as large as possible so the the sum of their indices is minimised. If any member of $\mathcal{R}$ is core-free, then the other members of $\mathcal{R}$ are superfluous, so in fact $\mathcal{R}$ is then transitive.

The study of this topic dates back to Johnson [2] where he proved that one can construct a minimal faithful representation $\{G_1, \ldots, G_n\}$ consisting entirely of so called *primitive* groups. These are groups which cannot be expressed as the intersection of groups that properly contain them.

We give a few examples of calculating the minimal degree when we have full access to the subgroup lattice.

Example 1. Let $G = C_{p^m}$ the cyclic group of order p^m where p is a prime number and m an non-negative integer. Then the lattice of subgroups forms a chain so the identity subgroup is the only core-free subgroup of G. For example, the lattice of subgroups for C_{p^3} is shown in Figure 2.

Figure 2. $\mathcal{L}(C_{p^3})$

It follows that the minimal faithful representation for any cyclic group of prime power order p^m is given by the identity subgroup and so $\mu(G) = p^m$.

Example 2. Let $G = D_4$ the dihedral group of order 8. Then its lattice of subgroups is as shown in Figure 3, where the normal subgroups are represented by filled dots and each edge represents a subgroup of index two.

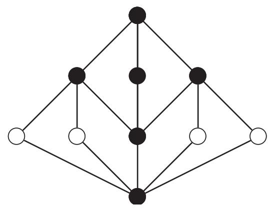

Figure 3. $\mathcal{L}(D_4)$

Suppose $\mathcal{R}$ is a minimal faithful collection of subgroups of D_4. By examining the lattice of subgroups, if $\mathcal{R}$ contained a normal subgroup and $\mathcal{R}$ did not contain the trivial subgroup, then it would also have to contain a non-normal subgroup. All non-normal subgroups are core-free of index 4 and so $\mathcal{R}$ is transitive. Therefore $\mu(D_4) = 4$.

Example 3. Let $G = Q_8 = \langle x, y \mid x^4 = 1, x^2 = y^2, x^y = x^{-1} \rangle$, the quaternion group of order 8. Its lattice of subgroups is shown in Figure 4 and all subgroups are normal.

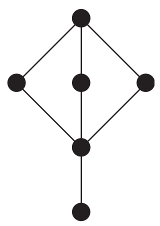

Figure 4. $\mathcal{L}(Q_8)$

Thus Q_8, like cyclic groups of prime power order, has a unique minimal normal non-trivial subgroup. Hence, any minimal faithful collection of subgroups for Q_8 must consist of only one subgroup, namely the identity. Therefore $\mu(Q_8) = 8$.

Examples 1 and 3 are scenarios where the minimal faithful degree of the group G is given by the *Cayley* or *regular* representation; that is, representing the group G acting on itself by right multiplication. Johnson [2, Theorem 1] classifies all cases where this occurs. Note that the four-group also has a minimal faithful representation which is not transitive.

Theorem 1. *The regular representation of a group G is minimal if and only if G is*

- *cyclic group of prime power order,*
- *a generalised quaternion two-group, or*
- *the four-group.*

The groups $G(m, p, n)$

In this section we follow the notation of [6].

Let m and n be positive integers, let C_m be the cyclic group of order m and $B = C_m \times \cdots \times C_m$ be the direct product of n copies of C_m. For each divisor p of m define the group $A(m, p, n)$ by

$$A(m, p, n) = \{(\theta_1, \theta_2, \ldots, \theta_n) \in B \mid (\theta_1\theta_2 \ldots \theta_n)^{m/p} = 1\}.$$

It follows that $A(m, p, n)$ is a subgroup of index p in B and the symmetric group $Sym(n)$ acts naturally on $A(m, p, n)$ by permuting the coordinates.

$G(m, p, n)$ is defined to be the semidirect product of $A(m, p, n)$ by $Sym(n)$. It follows that $G(m, p, n)$ is a normal subgroup of index p in the wreath product $C_m \wr Sym(n) = \underbrace{(C_m \times \cdots \times C_m)}_{n \text{ times}} \rtimes Sym(n)$ and thus has order $m^n n!/p$.

It is well known that these groups can be realised as finite subgroups of $GL_n(\mathbb{C})$, specifically as $n \times n$ matrices with exactly one non-zero entry, which is a complex m-th root of unity, in each row and column such that the product of the non-zero entries is a complex (m/p)th root of unity. Thus the groups $G(m, p, n)$ are sometimes referred to as monomial reflection groups. For more details on the groups $G(m, p, n)$, see [3], [1].

Direct products

The primary motivation for the author studying these monomial reflection groups is due to one of the central themes of Johnson [2] and Wright [7]. While it is clear that for any two finite groups G and H,

$$\mu(G \times H) \leq \mu(G) + \mu(H), \tag{1}$$

Johnson and Wright investigated under what conditions equality holds in (1). Johnson [2] proved that equality in (1) holds whenever G and H have coprime orders and Wright [7] proved that equality holds whenever G and H are p-groups and hence nilpotent groups.

Wright went further with this investigation constructing a class of finite groups $\mathcal{C}$ with the property that for any group $G \in \mathcal{C}$, G has a nilpotent subgroup G_1 such that $\mu(G_1) = \mu(G)$. It can easily be seen that $\mathcal{C}$ is closed under taking direct products and so any two groups in $\mathcal{C}$ yield an equality in (1). For if G and H are elements of $\mathcal{C}$, then

$$\mu(G) + \mu(H) = \mu(G_1) + \mu(H_1) = \mu(G_1 \times H_1) \leq \mu(G \times H),$$

and so $\mu(G \times H) = \mu(G) + \mu(H)$ and we can take $(G \times H)_1 = G_1 \times H_1$.

Wright proved that this class $\mathcal{C}$ contains all nilpotent, symmetric, alternating and dihedral groups, however the extent of this class is still unknown. At the end of his paper and in Johnson's paper, they both pose the question:

When is $\mu(G \times H) < \mu(G) + \mu(H)$ for two finite groups G and H?

Wright even asks whether equality is true for all finite groups. The referee to Wright's paper provided an example of when strict inequality holds and attached it as an addendum. The example was of degree 15 and involved the group $G(5,5,3)$, though this group was simply given in terms of permutations on a set of 15 letters.

It was observed by the referee that $G(5,5,3)$ has minimal degree 15 and moreover possesses a non-trivial centraliser in $Sym(15)$ which is isomorphic to C_5 and intersects trivially with it. Therefore

$$\mu(G(5,5,3)) = \mu(G(5,5,3) \times C_{Sym(15)}(G(5,5,3))) = 15$$

and so we immediately have a strict inequality to (1) by taking G and H to be $G(5,5,3)$ and $C_{Sym(15)}(G(5,5,3))$ respectively.

In [4], the author proved that a similar result occurs with the groups $G(4,4,3)$ and $G(2,2,5)$, that is $\mu(G(4,4,3)) = \mu(G(4,4,3) \times C_{Sym(12)}(G(4,4,3))) = 12$ and $\mu(G(2,2,5)) = \mu(G(2,2,5) \times C_{Sym(10)}(G(2,2,5))) = 10$, and so we obtain two more examples of strict inequality in (1). The author does not know whether 10 is the smallest degree for which strict inequality occurs.

Further, in [5] the author proved that for p and q distinct odd primes

$$\mu(G(p,p,q)) = \mu(G(p,p,q) \times C_{Sym(pq)}(G(p,p,q))) = pq$$

except when $p \equiv 1 \bmod 3$, thus demonstrating that the groups $G(p,p,q)$ provide an infinite family of examples for when strict inequality holds in (1). In the next section, we give a brief outline to the proof of this result; for more details and explicit proofs, see [5].

$\mu(G(p,p,q))$ for $p > q$

In this section we denote $G(p,p,q)$ by G and $A(p,p,q)$ by A throughout. We assume p and q are odd primes such that $p > q$ and exploit the action of $Sym(q)$ on A to prove that every minimal faithful representation of G is given by a core-free subgroup.

Observe that we may treat A as a semi-simple $Sym(q)$-module of dimension $q-1$ over the finite field $\mathbb{F}_p$ since p does not divide the order of $Sym(q)$. The following is a well-known result from modular representation theory.

Proposition 1. *$Sym(q)$ acts irreducibly and faithfully on A.*

Proof. We show that the submodule generated by an arbitrary non-trivial element is the whole of A. Let $w = \prod_{i=1}^{q} \theta_i^{\lambda_i}$ be a non-trivial element of A so that $\sum_{i=1}^{q} \lambda_i = 0$. It is enough to prove that we can obtain the basis elements $c_1 = \theta_1\theta_2^{-1}, \ldots, c_{q-1} = \theta_{q-1}\theta_q^{-1}$ of A via the action of $Sym(q)$ on w.

Fix a non-zero λ_i. There is another non-zero λ_j such that $\lambda_i - \lambda_j \neq 0$. For suppose $\lambda_i = \lambda_k$ for all non-zero λ_k. Then

$$w = \left(\prod_{j\in I} \theta_j\right)^{\lambda_i},$$

where I is a subset of $\{1, 2, \ldots, q\}$. So $\sum_{j \in I} \lambda_i = |I|\lambda_i = 0$ in $\mathbb{F}_p$. However since $p > q$, this implies that $\lambda_i = 0$, a contradiction.

Choose two non-zero λ_i and λ_j with $\lambda_i - \lambda_j \neq 0$. Then applying the transposition $(i\ j)$ to w we have

$$w^{(i\ j)} = \theta_1^{\lambda_1} \ldots \theta_i^{\lambda_j} \ldots \theta_j^{\lambda_i} \ldots \theta_q^{\lambda_q},$$

so

$$w(w^{(i\ j)})^{-1} = \theta_i^{\lambda_i - \lambda_j} \theta_j^{\lambda_j - \lambda_i} = (\theta_i \theta_j^{-1})^{\lambda_i - \lambda_j}.$$

Therefore, $\theta_i \theta_j^{-1}$ is contained in A and by applying the appropriate permutation to it, we can obtain all the basis elements $c_1, \ldots, c_{q-1}$ as required. So $Sym(q)$ acts irreducibly on A.

Now suppose that the action of $Sym(q)$ on A has a kernel. This kernel must be a normal subgroup of $Sym(q)$ and since $q \neq 4$, the only possibility is the alternating group $Alt(q)$. However, it can easily be verified that the q-cycle $b = (1\ 2\ \ldots\ q)$, which is an even permutation, does not commute with any non-trivial element of A. Therefore $Sym(q)$ acts faithfully on A.

Corollary 1. *A is the unique minimal normal subgroup of G.*

Proof. Certainly A is a normal subgroup of G and since $Sym(q)$ acts irreducibly on it, it is a minimal normal subgroup.

Suppose N is a non-trivial normal subgroup of G which does not contain A. By minimality of A we must have $A \cap N = \{1\}$. It follows that $AN = A \times N$, that is AN is the internal direct product of A and N.

Let $a \in A$ and $n = a'\sigma \in N \setminus A$, where $a' \in A$ and $\sigma \in Sym(q)$. Then $n = a^{-1}na$ so $a'\sigma = a^{-1}a'\sigma a = a'a^{-1}\sigma a$, and so $\sigma = a^{-1}\sigma a$. That is, σ commutes with a. But a is arbitrary so σ is contained in the kernel of the action of $Sym(q)$ on A. Therefore σ is trivial and $n = a'$ contradicting that $n \notin A$.

Therefore A is contained in every non-trivial normal subgroup of G and is thus the unique minimal normal subgroup of G.

It follows now that any minimal faithful representation of G must be transitive, that is, given by a single core-free subgroup. We use this fact to prove the minimal degree of G is pq.

Let L be a core-free subgroup of G such that $|G\colon L| = \mu(G)$. Since A is an elementary Abelian p-group of rank $q-1$, $\mu(A) = p(q-1)$ and since G is a proper subgroup of the wreath product $C_p \wr Sym(q)$ which has minimal degree pq, we have the upper and lower bounds

$$p(q-1) \leq \mu(G) \leq pq.$$

Via some arguments in linear representation theory involving duality, (see [5]) we can in fact prove (for $p \not\equiv 1 \bmod 3$) that any core-free subgroup of G has index at least pq and so $\mu(G) = pq$.

For the case $q = 3$ and $p \equiv 1 \bmod 3$ the calculation is easier. Observe that in this case, the group $G(= G(p,p,3))$ is isomorphic to $(C_p \times C_p) \rtimes Sym(3)$. Let c_1, c_2

generate the base group A and $a = (1\ 2), b = (1\ 2\ 3)$ generate $Sym(3)$. Then b and a act on the base group A as follows:

$$c_1^a = c_1^{-1}, \quad c_2^a = c_1c_2, \quad c_1^b = c_2, \quad c_2^b = c_1^{-1}c_2^{-1},$$

and this action induces a two dimensional $Sym(3)$-module structure on A. It is well known that when $p \equiv 1 \bmod 3$, there is a cube root of unity ζ_3 in the field $\mathbb{F}_p$. Observe that $\zeta_3^2 + \zeta_3 + 1 = 0$. Consider the element $c_1c_2^{-\zeta_3}$. We have

$$(c_1c_2^{-\zeta_3})^b = c_1^{\zeta_3}c_2^{\zeta_3+1} = (c_1c_2^{-\zeta_3})^{\zeta_3},$$

so $c_1c_2^{-\zeta_3}$ is an eigenvector for b with eigenvalue ζ_3. It is easily verified that $c_1c_2^{-\zeta_3}$ is not an eigenvector for a and so the subgroup $L = \langle c_1c_2^{-\zeta_3}, b\rangle$ forms a core-free subgroup of G of order $3p$. Since G has order $6p^2$, we have $|G\colon L| = 2p$, so $\mu(G) = 2p$.

Combining this with the previous arguments we have proved:

Theorem 2. *Let p and q be odd primes with $p > q$. Then*

$$\mu(G(p,p,q)) = \begin{cases} pq & \text{if } q \geq 5, \text{ or } q = 3 \text{ and } p \equiv 2 \bmod 3 \\ 2p & \text{if } q = 3 \text{ and } p \equiv 1 \bmod 3. \end{cases}$$

Acknowledgements

The author thanks his supervisor David Easdown and his associated supervisor Anthony Henderson for many helpful discussion regarding reflection groups and finite groups.

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From Lalescu's sequence to a Gamma function limit

Ovidiu Furdui*

Abstract

In this article we study a generalisation of Lalescu's sequence involving the Gamma function.

Introduction

Traian Lalescu, a great Romanian mathematician, has proposed in *Gazeta Matematica* [4] the study of the sequence $(L_n)_{n\in N}$ with the general term:

$$L_n = \sqrt[n+1]{(n+1)!} - \sqrt[n]{n!}.$$

The sequence $(L_n)_{n\in N}$, also known as Lalescu's sequence, has been studied by many Romanian mathematicians and it has been shown that it converges to $1/e$. For a recent study of this sequence as well as related Lalescu-like sequences, we recommend [3] and [5]. In this article we consider the following generalisation of $(L_n)_{n\in N}$ involving the Gamma function.

Theorem 1. *Let $m \geq 0$ be a natural number and let f be a polynomial of degree m whose coefficient in the leading term is positive. Then,*

(a) $$\lim_{x\to\infty}((f(x+2)\Gamma(x+2))^{1/(1+x)} - (f(x+1)\Gamma(x+1))^{1/x}) = \frac{1}{e}.$$

(b) $$\lim_{x\to\infty} x\left((f(x+2)\Gamma(x+2))^{1/(1+x)} - (f(x+1)\Gamma(x+1))^{1/x} - \frac{1}{e}\right) = \frac{1}{e}\left(m+\frac{1}{2}\right).$$

Before proving the theorem, we collect some known results about the Gamma function. We need, in our analysis, the following limit

$$\lim_{x\to\infty}\frac{(\Gamma(x+1))^{1/x}}{x} = \frac{1}{e}, \tag{1}$$

which can be proved by an application of Stirling's formula.

The psi function, also known as digamma function, is defined by $\psi(x) = \Gamma'(x)/\Gamma(x)$, and its derivative verifies the following asymptotic expansion, [1]:

$$\psi'(z) \approx \frac{1}{z} + \frac{1}{2z^2} + \frac{1}{6z^3} - \frac{1}{30z^5} + \cdots, \qquad z\to\infty, \quad |\arg z| < \pi. \tag{2}$$

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It follows, based on (2), that

$$\lim_{x\to\infty} x(x\psi'(x+1)-1) = -\frac{1}{2}\,. \tag{3}$$

Anderson [2] proved that if $h(x) = x\psi(x+1) - \ln\Gamma(x+1)$, then

$$\frac{h(x)}{x} = 1 + O\left(\frac{\ln x}{x}\right). \tag{4}$$

Now we are ready to prove Theorem 1. To make the calculations easier to follow we will write $\sqrt[x]{f(x)}$ instead of $(f(x))^{1/x}$.

Proof of Theorem 1

Proof. (a) Let f be a polynomial of degree m whose coefficient in the leading term is a positive real number. It follows that f is positive for large values of x and $(f(x))^{1/x}$ is well defined. We will prove that

$$\lim_{x\to\infty} (f(x+2)\Gamma(x+2))^{1/(1+x)} - (f(x+1)\Gamma(x+1))^{1/x} = \frac{1}{e}\,. \tag{5}$$

Let $u(x) = \sqrt[x]{\Gamma(x+1)f(x+1)}$ and $V(x) = u(x+1) - u(x)$. The mean value theorem implies that $V(x) = u'(c)$ for some $c \in (x, x+1)$. Thus, to prove (5) it suffices to show that $\lim_{x\to\infty} u'(x) = 1/e$. A straightforward calculation shows that,

$$\begin{aligned} u'(x) = {} & \frac{\sqrt[x]{\Gamma(x+1)}}{x}\sqrt[x]{f(x+1)} \\ & \times \left(\frac{\psi(x+1)x - \ln\Gamma(x+1)}{x} + \frac{x(f'(x+1)/f(x+1)) - \ln f(x+1)}{x}\right). \end{aligned}$$

On the other hand, since f is a polynomial, we obtain that

$$\lim_{x\to\infty} \sqrt[x]{f(x+1)} = 1 \quad \text{and} \quad \lim_{x\to\infty} \frac{x(f'(x+1)/f(x+1)) - \ln f(x+1)}{x} = 0\,. \tag{6}$$

Combining (1), (4) and (6) we get that $\lim_{x\to\infty} u'(x) = 1/e$.

(b) Let $L = \lim_{x\to\infty} x((f(x+2)\Gamma(x+2))^{1/(1+x)} - (f(x+1)\Gamma(x+1))^{1/x} - 1/e)$. An application of l'Hopital's rule shows that

$$\begin{aligned} L &= \lim_{x\to\infty} \frac{u(x+1) - u(x) - 1/e}{1/x} \\ &= \lim_{x\to\infty} \frac{u'(x+1) - u'(x)}{-1/x^2} \\ &= -\lim_{x\to\infty} x^2(u'(x+1) - u'(x)). \end{aligned}$$

Using the mean value theorem we obtain that $u'(x+1) - u'(x) = u''(c)$ for some $c \in (x, x+1)$, so $L = -\lim_{x\to\infty} x^2 u''(x)$. We note that,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\sqrt[x]{\Gamma(x+1)} &= \sqrt[x]{\Gamma(x+1)}\ \frac{\psi(x+1)x - \ln\Gamma(x+1)}{x^2}\\ &= \sqrt[x]{\Gamma(x+1)}\ a(x)\\ &= \sqrt[x]{\Gamma(x+1)}\ \frac{h(x)}{x^2}\,,\end{aligned}$$

where

$$a(x) = \frac{\psi(x+1)x - \ln\Gamma(x+1)}{x^2} \quad \text{and} \quad h(x) = \psi(x+1)x - \ln\Gamma(x+1).$$

Similarly,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\ \sqrt[x]{f(x+1)} &= \sqrt[x]{f(x+1)}\ \frac{(f'(x+1)/f(x+1))x - \ln f(x+1)}{x^2}\\ &= \sqrt[x]{f(x+1)}\ b(x)\\ &= \sqrt[x]{f(x+1)}\ \frac{g(x)}{x^2}\,,\end{aligned}$$

where

$$b(x) = \frac{(f'(x+1)/f(x+1))x - \ln f(x+1)}{x^2}$$

and

$$g(x) = \frac{f'(x+1)}{f(x+1)}\ x - \ln f(x+1)\,.$$

A calculation shows that,

$$a'(x) = \frac{\psi'(x+1)}{x} - \frac{2a(x)}{x} = \frac{\psi'(x+1)}{x} - \frac{2h(x)}{x^3}$$

and

$$b'(x) = \frac{(f'(x+1)/f(x+1))'}{x} - \frac{2g(x)}{x^3}.$$

It follows, based on formula $(fg)'' = f''g + 2f'g' + fg''$, that

$$\begin{aligned}u''(x) &= \sqrt[x]{\Gamma(x+1)}\,\sqrt[x]{f(x+1)}((a(x)+b(x))^2 + a'(x) + b'(x))\\ &= \sqrt[x]{\Gamma(x+1)}\,\sqrt[x]{f(x+1)}\\ &\quad\times\left(\frac{(h(x)+g(x))^2}{x^4} + \frac{\psi'(x+1)}{x} - \frac{2h(x)}{x^3}\right.\\ &\qquad\left. + \frac{(f'(x+1)/f(x+1))'}{x} - \frac{2g(x)}{x^3}\right).\end{aligned}$$

Thus, $x^2u''(x)$ is equal to

$$\begin{aligned}&\frac{\sqrt[x]{\Gamma(x+1)}}{x}\,\sqrt[x]{f(x+1)}\\ &\quad\times\left(\frac{(h(x)+g(x))^2}{x} + x^2\psi'(x+1) - 2h(x) - 2g(x) + x^2\left(\frac{f'(x+1)}{f(x+1)}\right)'\right).\end{aligned}$$

Therefore we obtain that,

$$L = -\frac{1}{e}\lim_{x\to\infty}\Bigg(\frac{(h(x)+g(x))^2}{x} + x^2\psi'(x+1) - 2h(x) - 2g(x) \\ + x^2\left(\frac{f'(x+1)}{f(x+1)}\right)'\Bigg). \tag{7}$$

On the other hand, a calculation shows that,

$$\lim_{x\to\infty} x^2\left(\frac{f'(x+1)}{f(x+1)}\right)' = -m. \tag{8}$$

Combining (3), (7) and (8) we obtain that

$$L = \frac{1}{e}\left(\frac{1}{2}+m\right) - \frac{l}{e},$$

where

$$\begin{aligned} l &= \lim_{x\to\infty}\left(\frac{(h(x)+g(x))^2}{x} - 2h(x) - 2g(x) + x\right) \\ &= \lim_{x\to\infty} x\left(\frac{h^2(x)+2h(x)g(x)+g^2(x)}{x^2} - \frac{2h(x)}{x} - \frac{2g(x)}{x} + 1\right). \end{aligned}$$

Using (4) we obtain that

$$\begin{aligned} l &= \lim_{x\to\infty} x\left(\left(\frac{h(x)}{x}-1\right)^2 + \frac{2g(x)}{x}\left(\frac{h(x)}{x}-1\right) + \frac{g^2(x)}{x^2}\right) \\ &= \lim_{x\to\infty}\left(xO^2\left(\frac{\ln x}{x}\right) + 2g(x)O\left(\frac{\ln x}{x}\right) + \frac{g^2(x)}{x}\right) = 0. \end{aligned}$$

The last equality is justified by the following calculations

$$xO^2\left(\frac{\ln x}{x}\right) = \frac{\ln^2 x}{x}\left[\frac{O(\ln x/x)}{\ln x/x}\right]^2 \to 0,$$

$$2g(x)O\left(\frac{\ln x}{x}\right) = 2\frac{O(\ln x/x)}{\ln x/x}\left(\frac{xf'(x+1)}{f(x+1)}\frac{\ln x}{x} - \ln f(x+1)\frac{\ln x}{x}\right) \to 0,$$

and

$$\frac{g^2(x)}{x} = \frac{1}{x}\left(x\frac{f'(x+1)}{f(x+1)}\right)^2 - 2x\frac{f'(x+1)}{f(x+1)}\frac{\ln f(x+1)}{x} + \frac{\ln^2 f(x+1)}{x} \to 0.$$

Therefore we obtain that $L = \frac{1}{e}(m+\frac{1}{2})$ and the theorem is proved.

Corollary 2. *Let f and g be polynomials of degree m and n whose leading coefficients are positive real numbers. Then,*

$$\lim_{x\to\infty} \frac{(f(x+2)\Gamma(x+2))^{1/(1+x)} - (f(x+1)\Gamma(x+1))^{1/x} - 1/e}{(g(x+2)\Gamma(x+2))^{1/(1+x)} - (g(x+1)\Gamma(x+1))^{1/x} - 1/e} = \frac{2m+1}{2n+1}.$$

Corollary 3. *Let f and g be polynomials of degree m and n whose leading coefficients are positive real numbers and let m, n, p and q be nonnegative integers such that $p \neq q$. Then,*

$$\lim_{x\to\infty} \frac{\begin{array}{l}(f(x+m+1)\Gamma(x+m+1))^{1/(m+x)} \\ \qquad - (f(x+n+1)\Gamma(x+n+1))^{1/(x+n)} - ((m-n)/e)\end{array}}{\begin{array}{l}(g(x+p+1)\Gamma(x+p+1))^{1/(p+x)} \\ \qquad - (g(x+q+1)\Gamma(x+q+1))^{1/(x+q)} - ((p-q)/e)\end{array}} = \frac{(2m+1)(m-n)}{(2n+1)(p-q)}.$$

Proof. Let

$$a_m(x) = (f(x+m+1)\Gamma(x+m+1))^{1/(m+x)}$$

and

$$b_p(x) = (g(x+p+1)\Gamma(x+p+1))^{1/(p+x)}.$$

We have that,

$$\begin{aligned}&\frac{a_m(x) - a_n(x) - (m-n/e)}{b_p(x) - b_q(x) - (p-q/e)} \\ &\quad = \frac{(a_m(x) - a_{m-1}(x) - (1/e)) + \cdots + (a_{n+1}(x) - a_n(x) - (1/e))}{(b_p(x) - b_{p-1}(x) - (1/e)) + \cdots + (b_{q+1}(x) - b_q(x) - (1/e))}.\end{aligned} \tag{9}$$

Multiplying by x both the numerator and the denominator of (9) and taking the limit completes the proof.

The following corollary gives a generalisation of Lalescu's sequence.

Corollary 4. *Let f be a polynomial of degree m with a positive leading coefficient. Then,*

(a) $$\lim_{n\to\infty} \left(\sqrt[n+1]{f(n+2)(n+1)!} - \sqrt[n]{f(n+1)n!} \right) = \frac{1}{e}.$$

(b) $$\lim_{n\to\infty} n \left(\sqrt[n+1]{f(n+2)(n+1)!} - \sqrt[n]{f(n+1)n!} - \frac{1}{e} \right) = \frac{1}{e}\left(m + \frac{1}{2}\right).$$

Remark 1. Letting $f = 1$ in the preceding corollary we obtain that the limit of Lalescu's sequence is $1/e$ and that the second term of the asymptotic expansion of L_n is $1/(2en)$, in other words

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} \left(\sqrt[n+1]{(n+1)!} - \sqrt[n]{n!} \right) = \frac{1}{e},$$

and

$$\lim_{n\to\infty} n\left(L_n - \frac{1}{e}\right) = \lim_{n\to\infty} n\left(\sqrt[n+1]{(n+1)!} - \sqrt[n]{n!} - \frac{1}{e}\right) = \frac{1}{2e}.$$

A natural question is to determine the asymptotic expansion of L_n.

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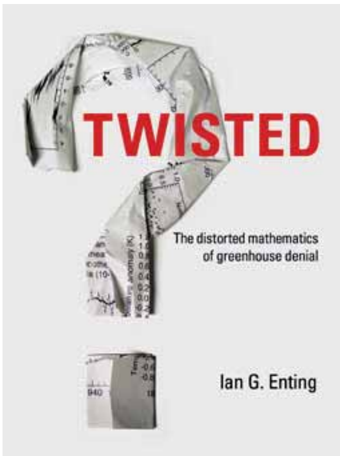
Book reviews

Twisted

Ian G. Enting
Australian Mathematical Sciences Institute, 2007, ISBN 978-0-6464-8012-1

Former CSIRO Division of Atmospheric Research scientist, Dr Ian Enting, presents an intriguing little book of just 152 pages including references, a glossary and various other aspects, aimed at exposing the contradictions in the arguments of the 'greenhouse sceptics'. In reviewing this book I kept pondering on the likely readership for such an essay. Would it be the 'greenhouse sceptics' themselves, I wondered? Not really — I would not think they would buy it . Rather, it may have readership in the undergraduate and lay community who are generally accepting of anthropogenic induced climate change. In other words, I doubt if this work would convince the hardened sceptics.

Rather, it is a concise book of armaments that the believer in anthropogenic-induced climate change can carry with them if ever confronted by a sceptic and driven to argument. In this, it provides useful responses to some of the currently popular sceptics' questions.



Enting avoids the inclusion of a lot of mathematical equations or anything unduly complex that would otherwise quickly lose readership in some sections of the community. This makes for a very easy and enjoyable read although more serious readers may want to look further afield if they are searching for more detailed input on this issue. I was a little disappointed by the relatively small amount of attention paid to the application of coupled general circulation models (GCMs). Five pages are devoted to this enormously important approach (including one page that contains a large flow diagram) which really is the key to provision of the likely outcomes, globally, of anthropogenic-induced climate change. In this respect, I'm not sure the quick and engaging responses provided in *Twisted* will totally disarm the entrenched sceptic. Enting follows Al Gore's approach in concentrating on interesting historical issues, which Enting does well, but perhaps at the expense of otherwise enlightening the reading audience on the outcomes associated with the fascinating area of science in climate and oceanographic modelling. Maybe others will take on this additional challenge now that Enting has provided us with a very useful formula by which to proceed.

One, finally, has to enjoy the insights Enting provides on the relative competencies, appropriate qualifications and backgrounds of some of the better-known sceptics. One suspects he could have driven further into this issue (and this aspect would have made for enticing reading!). For example, to be in charge of a meteorological or climatological office does not automatically qualify that person to be other than an administrator responsible for leave forms, rosters and human resources. Most of the population may not appreciate this fact regarding the management of scientific bureaucracies that Enting gently draws out. Similarly, a 'climatologist' may call themselves so but never have published on these issues or have qualifications associated with atmospheric or oceanographic circulation patterns at all. I found this to be one of the more insightful aspects in the pages of *Twisted*.

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A Mathematical Mosaic: Patterns & Problem Solving (Second Edition)

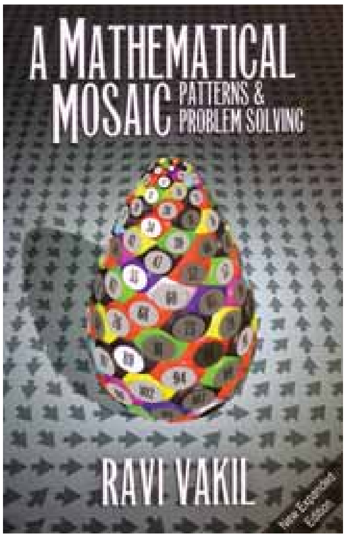
Ravi Vakil
Brendan Kelly Publishing, 2008, ISBN 978-1-8959-9728-6

To understand the author's objectives in writing this book it pays to know something about the author himself. As a teenager, Ravi Vakil represented Canada on three occasions at the International Mathematical Olympiad, the most elite mathematics competition for school students. He then went on to graduate school at Harvard University, during which time the first edition of *A Mathematical Mosaic* was penned and published. Since then, he has become an algebraic geometer of the highest calibre, with a rather impressive list of credentials and awards to his name. I have seen Vakil give extremely intelligible conference talks and this book portrays a little of that rare talent for technical communication.

A Mathematical Mosaic is essentially a platter of mathematical morsels, each occupying no more than a page or two. They have been carefully hand-picked so as to be easily digestible by motivated high school and early undergraduate students, whom the book is squarely aimed at. The vignettes are arranged into fifteen chapters ranging from broad headings like Number Theory or Geometry to more specific ones like Chessboard Colouring or Fibonacci and the Golden Mean. Vakil endeavours to serve them up while conveying the flavour of what it is to do mathematics and to be a mathematician. However, one can't help feel that some of the delight in discovering these morsels has been lost by having the meal so carefully prepared and organised. Some of these gems are probably better appreciated if discovered on one's own or, at least, within a greater context.

The topics discussed include some of the obligatory oldie-but-goodies such as the irrationality of $\sqrt{2}$, Euclid's proof of the infinitude of primes, and Archimedes' derivation of the volume of a sphere. Some less well-known facts concerning Pascal's triangle, Fibonacci numbers and combinatorial games also appear. Still other parts of the book are dedicated to ideas of a more recent vintage: the notion of cardinalities of sets, Arrow's theorem on voting systems, and a brief and ambitious attempt to discuss Galois theory.

Vakil expresses a sincere desire to stress that mathematics is not only about learning but also about doing and, to this end, there is a variety of problems littered throughout the book[1]. There are occasional historical diversions in the form of mini-biographies of mathematical greats such as Archimedes, Newton, Gauss and Ramanujan, among others. Accounts of the lives of Buckminster Fuller and Richard Feynman are also thrown in for good measure since they, too, embody the spirit of a mathematician. A more unusual and gratuitous inclusion is a series of portraits, outlining the development and achievements of eleven young people, all of whom have excelled at the International Mathematical Olympiad. This second edition has allowed Vakil to further track their career progress since the book debuted twelve years ago.



The main weakness of *A Mathematical Mosaic* is that it does not adequately highlight the rich tapestry that is mathematics. Rather, it merely exposes some of its various jewels, most in a superficial sort of way. On the flip side, it may be argued that one can only do so much in under three hundred pages of sparse text. In general, Vakil's writing style is extremely informal and, as a result, easily accessible. However, coupled with the accompanying cartoons, the character of the book treads the line between lighthearted and patronising. Furthermore, the typesetting is a little unsightly to a stickler like myself; hopefully, the pages will not be so disagreeable to other readers' eyes.

A reasonable proportion of those purchasing *A Mathematical Mosaic* will consist of highly motivated, overly ambitious school students, keen on retracing Ravi Vakil's glorious record in mathematics competitions. And they will most likely be disappointed, for the book certainly does not focus on any secret tricks or techniques for solving competition problems. Rather, Vakil's objectives are more admirable and it is certain that he will inspire fledgling mathematicians and entertain older ones through this book.

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[1]One example is the penny-in-a-corner problem which appears in Puzzle Corner 10 in this issue of the Gazette.

AMSI News

Philip Broadbridge*

Mathematical sciences and economic stability

Over the past month, world news has been dominated by the US-led economic downturn. We are reassured that Australian banks are not at such risk as those in US and UK, partly because we have a more stringent system of financial regulation and oversight. We at AMSI have some familiarity with this system because we have just completed a major review of the mathematical arrangements in PAIRS (Probability and Impact Rating System), commissioned by the Australian Prudential Regulation Authority. The project team consisted of William Dunsmuir and Scott Sisson (Statistics, University of New South Wales), Harald Scheule (Finance, University of Melbourne), and Dimetre Triadis, Tom Montague and myself (AMSI). Expressing a desire for transparency of their process, and inviting international feedback, APRA will be making the reports public.

Within the PAIRS process, experts of a specified financial sector (e.g. authorised deposit-taking institutions, general insurers, superannuation funds) assign numerical quality assessments on 18 factors such as counter party default risk, access to capital and expertise of senior management. These scores are combined within a quartic function to produce an index for overall risk of failure. Separately, an impact index is calculated, based on the total assets of the entity and a multiplier for extra community costs of failure. These two indices link to the SOARS (Supervisory Oversight and Response System) to determine the supervisory stance and regulations imposed.

Also, we looked over a proposed Benefit Model that estimates the financial benefits of increased regulatory intervention. We are satisfied that the APRA systems are serving us well. However, as mathematicians, we tried to identify weaknesses in its fabric. Contributing to the continuous improvement process, we identified some assumptions that needed more justification and we made a number of recommendations that should improve the mathematical consistency of the rating procedure without compromising its integrity.

The mathematical sciences contribute in many ways, both directly and indirectly, to economic stability. Actuarial calculations give insurers the confidence to protect companies and individuals against the financial consequences of accidents and calamities. Over the past twenty years, mathematicians have become much more involved in the rational pricing of financial derivatives. It could be argued that without the knowledge of pricing formulae, financial markets would have been even less stable. As we learned from last year's short course on electricity supply

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and pricing, against a background of fluctuating supply, energy retailers could not guarantee a future supply of energy at a modest price, were it not possible to hedge against suppliers' price volatility. We are already capable of making quantitative assessments of borrowers' and investors' financial capacities so that the widespread collapse of US-based mortgage schemes and merchant banks need not have occurred. Of course, government budgets cannot be composed without some predictions of macroeconomic variables over the forthcoming year. These predictions are informed by a multidimensional dynamical model. The way that this is done in Australia is reviewed by Hawkins [1].

A review of Nobel prizes in Economics shows that many of the awards are for advances in mathematical modelling. Most mathematicians are aware of the mathematical contributions of prize winners Scholes, Merton, Nash, Arrow and Samuelson. If only we could better model the human factors of greed, stupidity and lust for power or the short-sightedness of self-serving governments!

Economic development cannot be merely a movement of numbers from one account to another, or movement of chips from one square to another on a roulette wheel. Ultimately, national wealth requires value to be added to our resources, products and expertise. Economic stability requires us to have some control in adding value, not relying on externally determined prices of minerals. In this area, mathematical scientists could play an even greater role. We can maintain a competitive position in processing of raw materials and even maintain capability in strategic areas of manufacturing if we take care in creative design, efficient processing with high quality control and near-optimal scheduling, and delivery. All of these attributes require a high general level of mathematics and statistics. Several postgraduate students have now been placed in the AMSI industry internship scheme. We expect to be able to demonstrate the value of employing graduates who are well qualified in mathematics and statistics; demand for these skills is high but not as high as it should be.

References

[1] Hawkins J. (2005). Economic forecasting: history and procedures. *Economic Roundup*, Australian Government Treasury, Autumn 2005.

Director of AMSI since 2005, Phil Broadbridge was previously a professor of applied mathematics for 14 years, including a total of eight years as department chair at University of Wollongong and at University of Delaware.

His PhD was in mathematical physics (University of Adelaide). He has an unusually broad range of research interests, including mathematical physics, applied nonlinear partial differential equations, hydrology, heat and mass transport and population genetics. He has published two books and 100 refereed papers, including one with over 150 ISI citations. He is a member of the editorial boards of four journals and one book series.

News

General News

The CSIRO–ANZIAM student support scheme

To encourage students to attend conferences relevant to their interests, to emphasise the importance of students giving presentations about their work and to provide a first step educating students in ways of obtaining funding to help their research efforts, CSIRO and ANZIAM have initiated a student support scheme to provide funding to support conference travel and registration for students studying at universities in Australia and New Zealand, who wish to attend the ANZIAM conference or those of the ANZIAM special interest groups.

To be eligible, students will be required to present a talk at the conference and to be members of ANZIAM at the time of making their application. Funding will be at the discretion of the administering panel. For more details on the scheme and the application procedure see
http://www.anziam.org.au/The+CSIRO-ANZIAM+Student+Support+Scheme

Applications for this round for attendance at ANZIAM 2009 close on 19 December 2008.

Fermat Prize for mathematics research

The Fermat Prize rewards research works in fields where the contributions of Pierre de Fermat have been decisive:

- statements of variational principles;
- foundations of probability and analytical geometry;
- number theory.

The spirit of the Prize is focused on rewarding the results of researches accessible to the greatest number of professional mathematicians within these fields.

The amount of the Fermat Prize has been fixed at 20 000 Euros. The Fermat Prize is awarded once every two years in Toulouse; the tenth award will be announced in October 2009.

Winners of the preceding editions: A Bahri, K.A. Ribet (1989); J.-L. Colliot-Thélène (1991); J.-M. Coron (1993); A.J. Wiles (1995); M. Talagrand (1997); F. Bethuel, F. Hélein (1999); R.L. Taylor, W. Werner (2001); L. Ambrosio (2003); P. Colmez, J.F. Le Gall (2005); C. Khare (2007).

Rules governing the award, candidacy formalities, etc. are available from the organising secretariat of the Fermat Prize: Prix Fermat de Recherche en Mathématiques, Service Relations Publiques, Université Paul Sabatier, 31062 Toulouse Cedex 9, France, or on the web at http://www.math.ups-tlse.fr/Fermat/.

Closing date for application forms is 30 June 2009.

La Trobe University

Brian Davey has taken over from Peter Stacey as local correspondent. The editors of the *Gazette* thank Peter for his time as correspondent.

National curriculum for mathematics

The proposed national curriculum for mathematics has been discussed in the national press in recent days. If you wish to read more, see the discussion papers at http://www.ncb.org.au/consultation/expression_of_interest/national_forums.html. Click on the 'Get involved' heading to obtain directions for receiving notifications of updates to the site.

University of New South Wales

As mentioned in the last issue of the *Gazette*, the University of New South Wales Schools Mathematics Competition took place on 25 June this year. The winners have just been announced:

Competition Winners: Senior Division

Equal First Prize: Paul Cheung, Sydney Technical High School; Giles Gardam, Hurlstone Agricultural High School; Max Menzies, Sydney Grammar School.

Equal Second Prize: Sen Lin, James Ruse Agricultural High School; Irene Lo, James Ruse Agricultural High School.

Third Prize: Kevin Pan, James Ruse Agricultural High School.

Competition Winners: Junior Division

Equal First Prize: Raveen Daminha DeSilva, James Ruse Agricultural High School; Sampson Wong, James Ruse Agricultural High School.

Second Prize: Stacey Law, James Ruse Agricultural High School.

Equal Third Prize: Amelia Chowdhury, James Ruse Agricultural High School; Nancy Fu, Pymble Ladies' College; John Peter Wormell, Sydney Boys High School; Alan Xian, James Ruse Agricultural High School; Yi Zhang, James Ruse Agricultural High School.

University of Southern Queensland

The editors of the *Gazette* thank Dr Sergey Suslov for his time as local correspondent. Sergey is leaving to take up a position at Swinburne University of Technology in November.

Completed PhDs

University of Adelaide

- Dr Jeremy McMahon, *Time-dependence in Markovian decision processes*, supervisors: Nigel Bean, Michael Rumsewicz.
- Dr Selma Belen, *The behaviour of stochastic rumours*, supervisors: Charles Pearce, Tim Langtry.
- Dr Jens Kroeske, *Invariant bilinear differential pairings on parabolic geometrics*, supervisors: Michael Eastwood, Nick Buchdahl.
- Dr David Butler, *Quadrals and their associated subspaces*, supervisor: Sue Barwick.

University of Melbourne

- Dr Eugene Duff, *The detection and characterisation of complex spatio-temporal patterns of brain dynamics using fMRI, with an application to a study of motor learning*, supervisors: Ian Gordon, Gary Egan.

University of New South Wales

- Dr Danesh Jogia, *Algebraic aspects of integrability and reversibility in maps*, supervisor: John Roberts.

Awards and other achievements

Monash University

- Dr Simon Campbell won the 2008 Charlene Heisler Prize. The prize is awarded annually by the Astronomical Society of Australia for the most outstanding PhD thesis in astronomy or a closely related field. His thesis is entitled *Structural and nucleosynthetic evolution of metal-poor and metal-free low and intermediate mass stars.* Congratulations to Simon, and to his supervisor, Professor John Lattanzio.
- Dr Ian Wanless won a Young Tall Poppy Award.
- Congratulations to Professor Christian Jakob upon becoming a member of one of the Executive Council Task Teams in the World Meteorological Organization to devise a strategy for research into advanced weather and climate prediction.

University of Southern Queensland

- Dr Dmitry Strunin (together with Professor Tony Roberts, formerly USQ and now at the University of Adelaide), have been awarded an ARC Discovery Grant for the project 'Effective and accurate model dynamics, deterministic and stochastic, across multiple space and time scales', to the value of $315 000.

University of Sydney

- Neil Saunders has been awarded the Gordon Preston Prize for the best presentation given at the Victorian Algebra Conference by a current student based at an Australian university.

Appointments, departures and promotions

Flinders University

- Associate Professor Raj Huilgol has been promoted to Professor as from 1 July 2008.

La Trobe University

- From 1 August, James Atkinson has taken up a six-month post-doctoral position at the Department of Mathematics and Statistics. He will be working with Professor Reinout Quispel in the dynamical systems research group.

Monash University

- Dr Laura Davies commenced on 20 May 2008. Laura is a new Postdoctoral fellow working with Professor Christian Jakob in the Climate Group.
- Dr Richard Stancliffe, Research Fellow, commenced on 1 July 2008. His research interests are stellar evolution and nucleosynthesis, including binary stars and population synthesis.
- Dr John Mansour, Research Fellow, commenced on 21 July 2008. His research interests are applied mathematics, with particular interest in computational techniques for fluid dynamics.
- Dr Tianhai Tian, Senior Research Fellow, commenced on 11 August 2008. His area of research is in stochastic and multiscale modelling of biological systems, including genetic regulatory networks and cell signalling transduction pathways, stochastic simulation of biochemical reaction systems, computation in financial mathematics, numerical methods for stochastic differential equations, inference methods for estimating model parameters and parallel computing.
- Dr Christian Rau commenced on 1 September 2008 as a Lecturer. His areas of research are in multivariate analysis and statistics on manifolds, spatial statistics and geostatistics and stochastic geometry and image analysis.
- Dr Daniel Price commenced on 1 September 2008 as a Monash Fellow. His areas of research are broadly computational astrophysics: magnetohydrodynamics, self-gravitating gas dynamics and the smoothed particle hydrodynamics method.

Murdoch University

- Associate Professor Ken Harrison has retired after 32 years at Murdoch. He has been appointed Emeritus Associate Professor.

University of Melbourne

- Dr Maozai Tian has been appointed as Research Fellow.
- Dr David Wood has been appointed as QEII Fellow.
- Mr John Baldock has been appointed as MASCOS Network Administrator.
- Associate Professor William Blyth has been appointed as AMSI Research Fellow.
- Dr Gary Iliev has been appointed as MASCOS Research Fellow.
- Dr Jinghao Xue has been appointed as Research Fellow.

University of New England

- Dr Bea Bleile resumed as Associate Lecturer and Dr Imre Bokor resumed as Lecturer in September 2008.

University of New South Wales

- Dr Mohammad Akbar (from the University of Alberta) and Dr Rika Hagihara (from College of William and Mary) have begun working as postdoctoral fellows with Tony Dooley.
- Professor Fedor Sukochev has joined UNSW from Flinders University.

University of Queensland

- Dr Iadine Chades joins the Mathematics Department from the Institut National de la Recherche Agronomique (Toulouse). She has a 12-month appointment from July 2008 as Research Fellow working with Hugh Possingham and Phil Pollett on a project titled *Strategies for managing invasive species in space: deciding whether to eradicate, contain or control*, which is partially funded by ACERA, MASCOS and AEDA. She has research intersests in mathematical modelling and decision making in ecology.
- Dr Ross McVinish joins the Mathematics Department from QUT and has a three-year appointment as Research Fellow from September 2008 working with Phil Pollett. Ross has research experience in a number of areas of probability and statistics, from Lévy processes and stochastic processes displaying long memory, to Bayesian nonparametrics, and computation for Bayesian statistics and time series analysis.

University of Southern Queensland

- The Department of Mathematics and Computing has lost more staff than required through voluntary separation and resignation and will seek to advertise positions in mathematics and statistics shortly.

University of Sydney

- Dr William Bertram has resigned from his position as Lecturer in financial mathematics.
- Dr Georg Gottwald has been promoted to Principal Research Fellow/Associate Professor.

University of Western Australia

- Dr Tsoy-Wo Ma has taken up an Honorary Senior Lecturer position.
- Dr Csaba Schneider commenced six months as Research Fellow on a G08 European Fellowship.
- Dr Christian Thomas commenced as Research Associate.

New Books

Monash University

- Delbourgo, D. (2008). *Elliptic Curves and Big Galois Representations*. Cambridge University Press, London Mathematical Society Lecture Note Series 356, The London Mathematical Society.

Murdoch University

- Clarke, B. (2008). *Linear Models: The Theory and Application of Analysis of Variance*. Wiley Series in Probability and Statistics.

University of New England

- Bleile, B. (2008). *Poincaré Duality Pairs of Dimension Three*. VDM Verlag.

Conferences and Courses

Conferences and courses are listed in order of the first day.

Algebras, Operators and Noncommutative Geometry

Date: 1–5 December 2008
Venue: Centre for Mathematics and its Applications, ANU
Web: http://wwwmaths.anu.edu.au/events/aong08/

UNSW Research Workshop: Function Spaces and Applications

Date: 2–6 December 2008
Venue: University of New South Wales, Sydney
Web: http://web.maths.unsw.edu.au/~aji23/WFSA261208.html

The workshop is intended to bring together engineers and mathematicians to exchange problems, solutions and ideas and to foster both interdisciplinary and in-disciplinary research and collaboration. The focus of the workshop is on function spaces and their applications in the wide sense, including: functional and harmonic analysis and PDE; image and signal processing; Besov, Lizorkin–Triebel and Sobolev spaces; optimal computation problems, algorithms and complexity; functional calculus and vector-valued functions; wavelets, splines and stochastic models.

TRANSITIONS: a one-day workshop on mathematical curriculum issues in the transition from school to university

Date: 3 December 2008
Location: La Trobe University, Bundoora campus
Web: http://wt.maths.latrobe.edu.au/register/Conferences/Transitions/

7th Australia–New Zealand Mathematics Convention (ANZMC2008)

Date: 8–12 December 2008
Venue: Christchurch, New Zealand
Web: http://www.math.canterbury.ac.nz/ANZMC2008/

4ICC (4th International Conference on Combinatorial Mathematics and Combinatorial Computing)

Date: 15–19 December 2008 Venue: Auckland, New Zealand
Web: http://www.cs.auckland.ac.nz/research/groups/theory/4ICC/

Special theme program on group theory, combinatorics and computation

Date: 5–16 January 2009
Venue: The University of Western Australia, Perth
Web: http://sponsored.uwa.edu.au/gcc09/
Email: gcc09@maths.uwa.edu.au

2009 AMSI Summer School

Date: 12 January to 6 February 2009
Venue: University of Wollongong
Web: http://www.uow.edu.au/informatics/maths/summerschool/index.html

The AMSI Summer School is designed for Honours and coursework masters students in mathematical and statistical sciences. Courses will be available in the areas of applied mathematics, pure mathematics and statistics. The annual Mathematics in Industry Study Group will be held at Wollongong 27–31 January 2009 and this has been incorporated into one of the courses.

More details, including a registration form and subsidy information, are available on the website. The deadline for registration is 21 November 2008.

AMSI is funded by the Australian Government Department of Education, Employment and Workplace Relations and we thank them for their support.

CATS 2009. Computing: The Australasian Theory Symposium

Date: 20–23 January 2009
Venue: Victoria University of Wellington, New Zealand
Web: http://velorum.ballarat.edu.au/~pmanyem/CATS09

Celebrating Bob Anderssen's 70th birthday: ANU/CSIRO one-day seminar on computational mathematics and inverse problems

Date: 23 January 2009
Venue: ANU Manning Clarke Lecture Theatre 6 and Foyer, ANU
Web: http://wwwmaths.anu.edu.au/events/Bob70

Bob Anderssen has applied mathematics to a very broad range of problems ranging from the vibrations of the earth to the formation of patterns in plants. Two themes that have been threads that draw together much of this work are the application of computational mathematics and the analysis of inverse problems. This meeting, which focuses on computational mathematics and inverse problems and features speakers who have collaborated with Bob, will attempt to capture the diversity of his work in these areas.

Organising committee: Frank de Hoog, CSIRO; Markus Hegland, Australian National University.

There is no registration fee for attendance and everyone with an interest in mathematical modelling, computation and inverse problems, and friends of Bob are most welcome to attend. However, in order for us to arrange catering, please register through the website.

ANZIAM 2009

Date: 1–5 February 2009
Venue: Rydges Oasis Resort, Caloundra, Queensland
Web: http://www.sci.usq.edu.au/conference/index.php/ANZIAM/2009

We have added to our list of invited speakers. The list now reads: Prof Jim Hill (Wollongong, ANZIAM Medalist, 2008); Prof Ian Turner (QUT); Prof Kerrie Mengersen (QUT); Prof Phillip Maini (Oxford); Dr Graham Weir (Industrial Research Limited, NZ); Prof Guy Latouche (Bruxelles); Prof Natashia Boland (Newcastle); Prof Jerzy Filar (University of South Australia).

The closing date for registration and abstract submission is 19 December 2008.

HDA09: 3rd workshop on high-dimensional approximation

Date: 16–20 February 2009
Venue: School of Mathematics and Statistics, University of New South Wales
Web: http://conferences.science.unsw.edu.au/hda09/

Workshop on complex geometry

Date: 16–20 February 2009
Venue: Institute for Geometry and its Applications, University of Adelaide
Web: http://maths.adelaide.edu.au/~flarusson/workshop.html

Speakers (as of mid September): Nicholas Buchdahl, University of Adelaide; Emma Carberry, University of Sydney; Jean-Pierre Demailly*, Université de Grenoble;

Michael Eastwood, University of Adelaide; Peter Ebenfelt*, University of California, San Diego; Vladimir Ejov, University of South Australia; John Erik Fornaess, University of Michigan; Franc Forstneric*, University of Ljubljana; Rod Gover, University of Auckland; Adam Harris, University of New England; Alexander Isaev, Australian National University; Kang-Tae Kim, Pohang University of Science and Technology; Finnur Larusson, University of Adelaide; Jurgen Leiterer, Humboldt-Universitat; Morris Kalka, Tulane University; Amnon Neeman*, Australian National University; Paul Norbury, University of Melbourne; Gerd Schmalz, University of New England. (*Keynote speakers will give 2-3 lectures each, the first lecture being a survey intended for a broad audience.)

There is no formal registration process or fee, but please let us know if you plan to attend the workshop.

For more information, please contact one of the organisers: Nicholas Buchdahl, Michael Eastwood or Finnur Larusson at firstname.lastname@adelaide.edu.au.

The workshop is supported by AMSI.

Australia–Latin America Mining Summit 2009

Date: 23–27 February 2009
Venue: University of New South Wales
Web: http://www.complex.org.au/c_events.php

International workshop on modelling and data analysis for infectious disease control

Date: 9–12 March 2009
Venue: Murramarang, NSW
Web: http://nceph.anu.edu.au/News/modelling_workshop.php

The workshop title reflects the major theme, which includes assessment of vaccine effects, vaccination strategies, effectiveness of antiviral drugs, non-pharmaceutical interventions and methods to estimate the parameters and functions that inform these.

Interested persons are asked to complete an expression of interest and fax it to +61 2 6125 0740.

For program information, contact Niels Becker + 61 2 6125 4578. For practical information, contact Ros Hales +61 2 6125 5627.

NSDS09

Date: 22–27 June 2009
Venue: Sevilla, Spain
Web: http://congreso.us.es/nsds09

On the occasion of Peter Kloeden's 60th birthday, the International Conference on Non-autonomous and Stochastic Dynamical Systems and Multidisciplinary Applications (NSDS09) is being held in Sevilla (Spain) on 22–27 June 2009.

This conference aims to be a second edition of the NSDS05 which was held in Sevilla in 2005 (http://grupo.us.es/gaesdif/nsds05), and is also closely related to the conferences held in 2002 (http://wmatem.eis.uva.es/~dmde02/) and in 2007 (http://wmatem.eis.uva.es/~dm07/).

1st PRIMA Congress

Date: 6–10 July 2009
Venue: University of New South Wales, Sydney
Web: http://www.primath.org/prima2009
Contact: Alejandro Adem (adem@pims.math.ca)
Local Arrangements Committee (prima2009@maths.unsw.edu.au)

PRIMA (Pacific Rim Mathematical Association) is an association of mathematical sciences institutes, departments and societies from around the Pacific Rim. It was established in 2005 to promote and facilitate the development of the mathematical sciences throughout the Pacific Rim region. PRIMA aims to hold an international congress every four years.

As well as plenary addresses by leading international speakers there will be a range of special sessions on topics reflecting the breadth and diversity of research in the mathematical sciences across the region.

Third Japanese/Australian workshop on real and complex singularities

Date: 15-18 September 2009
Venue: The University of Sydney (Medical Foundation Auditorium)
Web: http://www.maths.usyd.edu.au:8000/u/laurent/RCSW

The first two Australian–Japanese Workshops on Real and Complex Singularities were held in Sydney in September 2005 and in Kyoto in November 2007. The third workshop will be held in Sydney in September 2009. All interested mathematicians are welcome to attend.

Vale

University of Sydney

It is with deep regret and sadness that the School of Mathematics and Statistics announces the sudden death of Dr Marc Raimondo, Senior Lecturer in Statistics, on 10 August 2008.

Visiting mathematicians

Visitors are listed in alphabetical order and details of each visitor are presented in the following format: name of visitor; home institution; dates of visit; principal field of interest; principal host institution; contact for enquiries.

Dr Florent Autin; Université Paris 7; 15 October 2008 to 15 December 2008; maxisets; USN; M. Raimondo

Prof Rosemary Bailey; University of London; 22 to 30 November 2008 and 7 to 20 December 2008; design of experiments; USA; Dr Chris Brien

Prof Thomas Bartsch; Giessen; 1 October 2008 to 15 November 2008; topological methods in nonlinear analysis; USN; E.N. Dancer

Prof Dorin Bucur; Universite de Savoie; 1 October 2008 to 30 November 2008; boundary value problems for partial differential equations; USN; E.N. Dancer

Prof Alain Bruguieres; University of Montpellier II; 1 November 2008 to 10 December 2008; Hopf algebras and category theory; MQU; Em/Prof Ross Street

Prof Dorin Bucur; Universite de Savoie; 1 October 2008 to 30 November 2008; boundary value problems for partial differential equations; USN; E.N. Dancer

Dr Nirmalendu Chaudhuri; University of Wollongong; 1 July 2008 to 31 December 2008; applied and nonlinear analysis; ANU; Neil Trudinger

Dr Florica Cirstea; University of Sydney; 14 July 2008 to 14 July 2011; applied and nonlinear analysis; ANU; Neil Trudinger

Prof David Clark; SUNY, New Platz, NY, USA; 15 October 2008 to 15 December 2008; universal algebra; LTU; Drs Brian A. Davey.
(Note: Professor Clark will be visiting the Research Group on General Algebra and its Applications and will be a Distinguished Fellow at La Trobe's Institute for Advanced Study for two months.)

Dr Robert Clark; University of Wollongong; 1 July 2008 to 1 July 2011; statistical science; ANU; Alan Welsh

Professor Garth Dales; University of Leeds; 2 to 24 October 2008; analysis and geometry; ANU; Rick Loy

Prof Gauri Datta; University of Georgia; 5 September to 5 November 2008; UMB; Prof Richard Huggins

Prof Zengji Du; School of Mathematical Sciences, Xuzhou Normal University; 22 October 2008 to 22 October 2009; differential equations; Chris Tisdell

Mr Ivan Dynov; Max-Planck Institut für Mathematik; 1 September 2008 to 31 December 2008; analysis and geometry; ANU; Alan Carey

Prof Ahmad Erfanian; University of Mashland, Iran; May to December 2008; UWA; Prof Cheryl Praeger

Em/Prof Christopher Field; Dalhousie University; 7 November 2008 to 3 December 2008; asymptotic methods in statistics; USN; J. Robinson

Prof Fereidoun Ghahramani; University of Manitoba; 10 November 2008 to 1 January 2009; analysis and geometry; ANU; Rick Loy

Professor Philip Griffin; Syracuse University; 1 February 2009 to 21 March 2009; financial mathematics; ANU; Ross Maller

Professor Hong Gu; Dalhousie University; 18 January 2009 to 28 February 2009; statistical science; ANU; Alan Welsh

Prof Zongming Guo; Henan Normal University; 31 August 2008 to 15 November 2008; large solutions of nonlinear elliptic partial differential equations; USN; E.N. Dancer

Prof Chong Chao Huang; Wuhan University, China; June 2008 to January 2009; –; UWA; A/Prof Song Wang

Dr Alexey Isaev; Bogoliubov Laboratory of Theoretical Physics, Russia; 1 October 2008 to 1 December 2008; quantum algebras, their symmetries, invariants and representations; USN; A.I. Molev

Dr Philip Kokic; ABARE; 8 July 2008 to 7 July 2009; statistical science; ANU; Alan Welsh

Dr Jacek Krawczyk; Victoria University of Wellington, New Zealand; 1 September 2008 to 7 November 2008; economic and environmental applications of control theory and dynamical systems; USA; Prof Jerzy Filar

Prof Tony Krzesinski; University of Stellenbosch, South Africa; 21 November 2008 to 23 December 2008; –; UMB; Prof Peter Taylor

Dr Philippe Lauret; University of La Reunion, Reunion Island; 8 August 2008 to 9 February 2009; mathematical and statistical modeling of energy systems; USA; A/Prof John Boland

Prof Charles Leedham-Green; Queen Mary and Westfield College; 30 December 2008 to 17 January 2009; –; UWA; Dr Alice Niemeyer

Prof Michael Leinert; University of Heidelberg; 30 October 2008 to 8 March 2009; harmonic analysis and Banach algebras; UNSW; Ian Doust

Prof Michael Leinert; University of Heidelberg; 1–28 December 2008; harmonic analysis; MDU; Prof Walter Bloom

Ms Nan Li; The Sichuan Normal University, China; 1 February 2008 to 31 January 2009; –; UWA; A/Prof Song Wang

Dr Martin Markl; Institute of Mathematics of the Academy of Sciences of the Czech Republic; 18 November 2008 to 15 December 2008; higher category theory; MQU; Dr Michael Batanin

Dr James McCoy; University of Wollongong; 1 January 2009 to 30 June 2009; applied and nonlinear analysis; ANU; Ben Andrews

Professor William Messing; University of Minnesota; 20 September 2008 to 29 October 2008; algebra and topology; ANU; Amnon Neeman

Dr Christine Mueller; University of Kassel, Germany; 1 October 2008 to 31 December 2008; –; UMB; Prof Richard Huggins

Dr Gernot Mueller; University of Technology, Munich; 25 October 2008 to 26 December 2008; financial mathematics; ANU; Ross Maller

Dr Alireza Nematollani; University of Shiraz; 15 December 2007 to 15 December 2008; multivariate analysis and time series; USN; N.C. Weber

Prof Shuangjie Peng; Central China Normal University; 15 June 2008 to 31 January 2009; nonlinear elliptic partial differential equations; USN; E.N. Dancer

Prof Helen Perk; Oklahoma State University; 1 August 2008 to 31 May 2009; mathematical physics; ANU; Murray Batchelor

Prof Jacques Perk; Oklahoma State University; 1 August 2008 to 31 May 2009; mathematical physics; ANU; Murray Batchelor

Prof Ulf Persson; Chalmers University; 1 November 2008 to 31 December 2008; algebra and topology; ANU; Amnon Neeman

Dr Pierre Portal; University of Lille; 23 February 2009 to 6 April 2009; analysis and geometry; ANU; Alan McIntosh

Prof. Sebastian Reich; University of Potsdam; 1 October 2008 to 12 December 2008; Multi-scale methods in science and engineering; USN; G. Gottwald

Alan Professor Steve Rosenberg; Boston University; 1 March 2009 to 1 July 2009; analysis and geometry; ANU; Alan Carey McIntosh

Dr Jerome Scherer; Universitat Autonoma de Barcelona; 3 September 2008 to 15 December 2008; algebra and topology; ANU; Amnon Neeman

Dr Qiao Shouhong; Sun Yat-sen University, China; 1 March 2008 to March 2009; –; UWA; A/Prof Cai Heng Li

Maryam Solary; Guilan University, Iran; July 2008 to January 2009; –; UWA; A/Prof Song Wang

Dr Damien Stehle; Ecole Normale Superieure, Lyon; 19 July 2008 to 18 July 2009; computational aspects of lattices; USN; J.J. Cannon

Prof Ralph Stohr; University of Manchester; 21 September 2008 to 10 December 2008; algebra and topology; ANU; Laci Kovacs

Prof Alexis Virelizier; University of Montpellier II; 1 November 2008 to 10 December 2008; Hopf algebras and category theory; MQU; Em/Prof Ross Street

Dr Huoxiong Wu; Xiamen University, China; January 2008 to November 2008; harmonic analysis and partial differential equations; MQU; X.T. Duong

Professor Chengmin Zhang; Chinese Academy of Science; 15 November 2008 to 15 December 2008; astronomy and astrophysics; ANU; Lilia Ferrario

AustMS

AustMS Accreditation

The secretary has announced the accreditation of:

- Professor K. A. Smith-Miles of Deakin University, as an Accredited Fellow (FAustMS).

The ANZIAM/CSIRO student support scheme for attendance at the ANZIAM and special interest group conferences

The ANZIAM/CSIRO Student Support Scheme (SSS) is designed to provide funding to support conference travel and registration for students studying at universities in Australia and New Zealand, who wish to attend the ANZIAM conference or those of the special interest groups. The scheme will be funded by the annual sponsorship that has been provided to ANZIAM by CSIRO, together with any supplementary funding that the ANZIAM Executive may wish to vote.

The objectives of the SSS are to:

- help students attend conferences relevant to their interests
- emphasise the importance of students giving presentations about their work
- provide a first step educating students in ways of obtaining funding to help their research efforts.

In order to be eligible for funding, students will be required to present a talk at the conference. It is expected that students will have discussed their plans with their supervisors, and supervisor endorsement of applications is required.

If students are not already members of ANZIAM, they must join at the time of making their application. The committee may wish to give preference to students who are already members of ANZIAM and whose supervisors are members of ANZIAM, who will be attending the same conference.

Background

ANZIAM believes that all research students in applied mathematics should be encouraged to attend conferences and present their results on a regular basis. The benefits to both the students and the applied mathematics community in general that derive from students having the opportunity to receive feedback on their work and to network with both senior colleagues and other students are invaluable.

The SSS has been designed to help students attend conferences and also to give them a relatively painless initial experience in making applications for funding.

The SSS is based on the principle that students should provide a complete budget for their travel, registration and accommodation, attempt to obtain partial funding from other sources and request the gap between what they can obtain elsewhere and the total amount that they need from the SSS.

Budget and operation of the scheme

The ANZIAM Executive will allocate a budget to the scheme each year, which shall at least include the designated component of the CSIRO contribution.

A panel of three ANZIAM members, including a CSIRO representative, shall be constituted to make decisions on the applications. The panel will have discretion to make decisions on each application, taking into account the individual circumstances of the students involved. Overall the panel will be responsible for treating applications fairly according to the principles of the scheme and spreading the budget out equitably over each of the eligible conferences.

Further information, including deadlines for each of the eligible conferences and the application procedure will appear on the society's website (http://austms.org.au) shortly.

The Australian Mathematical Society

Membership and Correspondence

Applications for membership, notices of change of address or title or position, members' subscriptions, correspondence related to accounts, correspondence about the distribution of the Society's publications, and orders for back numbers, should be sent to the Treasurer. All other correspondence should be sent to the Secretary. Membership rates and other details can be found at the Society web site: http://www.austms.org.au.

Local Correspondents

Publications

The Journal of the Australian Mathematical Society

Editor: Professor M. Cowling
School of Mathematics
University of Birmingham
Edgbaston, Birmingham B15 2TT
UK

The ANZIAM Journal

Editor: Professor C.E.M. Pearce
School of Mathematical Sciences
The University of Adelaide
SA 5005
Australia

Bulletin of the Australian Mathematical Society

Editor: Associate Professor D. Taylor
Bulletin of the Australian Mathematical Society
School of Mathematics and Statistics
The University of Sydney
NSW 2006
Australia

The *Bulletin of the Australian Mathematical Society* aims at quick publication of original research in all branches of mathematics. Two volumes of three numbers are published annually.

The Australian Mathematical Society Lecture Series

Editor: Professor C. Praeger
School of Mathematics and Statistics
The University of Western Australia
WA 6009
Australia

The lecture series is a series of books, published by Cambridge University Press, containing both research monographs and textbooks suitable for graduate and undergraduate students.

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