Increasing access to mathematical thinking

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Abstract

This article is a contribution to understanding of teacher actions that can contribute to a successful mathematics learning experience, defined as one that engages all students, especially those who may sometimes feel alienated from mathematics and schooling, in productive and successful mathematical thinking and learning. We offer an example of a task that can form the basis of such a learning experience. The key elements are that the task is open-ended, that the teacher offers specific pedagogical prompts to support student leaning, that the teacher builds a sense of community by ensuring that there are some common experiences, and the teacher prepare prompts that can be used to support students who are experiencing difficulty, or to extend those students who complete the basic task readily.
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1 Introduction

Our assumption is that most readers of this article love mathematics, enjoy teaching it, and mainly do so to develop willing and confident learners. In contrast, we know that the school experiences of many students are such that they choose not to pursue further studies in mathematics. This has the effect of reducing the range of courses that they can study as well as limiting their capacity to interpret life experiences from a mathematical perspective. Both school teachers and university lecturers have an interest in improving the mathematical experiences of students.

We are currently working on a project that is examining ways to maximise learning opportunities for students in upper primary and lower secondary mathematics classes. There are two aspects to the project: the type of tasks posed, and particularly ways to assist students experiencing difficulty with those tasks; and the steps that teachers can take to overcome structural barriers to mathematical learning by being explicit about classroom processes.

2 Using open-ended mathematical tasks to engage all students

An underlying premise to our approach to mathematics teaching is that engagement in mathematical thinking comes from students working on a succession of problem-like tasks, rather than following the teachers instructions, step by step. There are two parts to the rationale for this. The first is recognition that learning and knowing is a product of activity that is "individual and personal, and ... based on previously constructed knowledge" [1 §2]. The second relates to the role of the teacher in identifying blockages, prompts, supports, challenges and pathways. Cobb and McClain argued that teachers should have a clear impression of the direction that the learning of the individuals and the class will take. They proposed that the teacher should form an "instructional sequence (that) takes the form of a

conjectured learning trajectory that culminates with the mathematical ideas that constitute our overall instructional intent" [2 §24]. In other words, neither is it intended that students learn mainly from listening to teacher explanations, nor that they work solely on unstructured but potentially rich mathematical tasks. The learning occurs as a product of students working on tasks purposefully selected by the teacher, and contributing to ongoing dialog with the teacher and their peers on their strategies and products. Open-ended problems provide a suitable basis for achieving both of these goals.

First, some definitions. The *task* is the statement presented to students that serves as the prompt for their work. *Activity* refers to the thoughts and actions in which they engage in response to the prompt. The *goal* is the result the students seek as a product of their activity in response to the task statement. Each has the potential to be open or closed. Closed implies there is only one acceptable pathway, response, approach, or line of reasoning. Open refers to the existence of more than one (preferably many more than one) possible pathways, responses, approaches or lines of reasoning. We term tasks *open-ended* if they have open goals, and we prefer tasks that prompt open activity. (We note the common usage of the term *open problem* that refers to currently unsolved problems. Our use of the term open-ended is intended to distinguish the tasks we use from open problems.)

Open activity and open goals foster some of the more important aspects of learning mathematics, including investigating, creating, problematising, communicating, and thinking–as distinct from merely recalling procedures. This is evident in the example used below.

3 Developing communal understanding

A key aspect of our work, though, is on the ways that teachers can support students who experience difficulty with a task set that is set for the class. It is common for teachers to gather students experiencing difficulty together and teach them as a group, probably more slowly and perhaps in a louder voice and more repetition of procedural steps. We suggest that students are more likely to feel fully part of the class, and so be more likely to choose to participate fully, if teachers offer prompts to allow those experiencing difficulty to engage in active experiences related to the initial goal task, rather than, for example, requiring such students to listen to additional explanations or assuming that they will pursue goals substantially different from the rest of the class.

A further aspect relates to teachers anticipating that some students may complete the planned tasks quickly and posing supplementary tasks that extend their thinking on that same task, rather than proceeding onto the next lesson. Here, the aim is to move the learners forward, so that every student can engage in collective mathematical argumentation, reflection and dialogue and all will be ready for the subsequent lesson. The linguistic, psychological and cultural nature of classroom talk contributes to the development of deeper communal understanding of mathematical concepts and principles [3 §87].

One of the characteristics of open-ended tasks is that they create opportunities for extension of mathematical thinking, since students can explore a range of options as well as considering forms of generalised response. Another is the range of solution methods brought to many open-ended problems, with discussions about these having the potential to make methods that students use more powerful and efficient through communal consideration of similarities, differences and links between various solution processes, why particular methods work–some more efficiently than others–and also possibilities for further generalisation, abstraction and applications.

4 Being explicit about classroom processes

The second part of our research relates to structural barriers that may inhibit the learning of some students. There are factors inherent in the culture of schooling, particularly for some lower socio-economic status students, that may constrain the potential of teaching strategies to engage them in meaningful learning. Delpit argued that schools should actively seek to teach their usually *implicit* values, and that to pretend that schooling (and society) is democratic, for example, actually denies groups outside the mainstream access to the opportunities that schooling is intended to provide [4]. Zevenbergen similarly argued that teachers should make socio-cultural norms of pedagogy explicit to students [5]. For example, we suggest that teachers explicitly explain to students, inter alia, such issues as the forms of response sought, the potential for creativity, the mathematical focus and purpose of the task, the nature of communication sought, their criteria for evaluating responses, and particular terminology, especially mathematics terms.

5 The volume of boxes made from card: An example of a mathematical experience

The following task is used to illustrate the key aspects of our project. We would pose such a task to lower to middle secondary level classes. It is a simpler version of the more common problem of maximising the volume of a box created from rectangular card.

A closed version of the task would be:

> Squares of side 2 cm are cut from a rectangular sheet that is 20 cm long and 16 cm wide, and the resulting shape is folded into a box. What is the volume of the box?

The open-ended version of the task is:

> Suppose that you have a rectangular sheet that is 20 cm long and 16 cm wide, and you cut squares out of each of the corners, and then fold up the sides to make an open-top box. Calculate the volume of some boxes that can be made from that card.

We see the advantages of the open-ended form over the closed version are that it is able to be used with heterogeneous classes since students can approach the task at different levels and in different ways, and that this form of the task can focus students' attention onto key aspects of concepts (when different sized squares are cut out this results in different sized boxes, that the volume of these boxes varies, etc.) as distinct from them merely trying to recall and apply rules. This task also allows students opportunities to investigate the problem context, make decisions, generalise, seek connections, and identify alternatives.

We are aware that some readers will not see the latter task as open-ended since, by assigning a variable to the length of the side of the square, the volume of such boxes can be described readily. We assume the task will be posed to students at an age prior to them having an orientation to seeking general solutions to such tasks in this way. We anticipate that many lower to middle secondary students will seek numerical solutions, even restricting themselves to whole numbers. Nevertheless we anticipate that some students may explore the possibility of describing a general solution.

A teacher could well anticipate that some other students might experience difficulty with the task. We suggest that teachers not try to predict which students might experience difficulty but allow all students to commence working on the task and make decisions based on the students responses to the task. It is important to recognise that some students may be unfamiliar with some linguistic or procedural aspects of such tasks. For example, the

teacher could emphasise the meaning of key terms, such as *volume* and *open-top* box, could explain that there are multiple possible solutions and many different ways of representing solutions, that creativity is desirable, that responses do not need to presented neatly in the first instance, that it can help if they imagine what the box might look like, and so on.

Most importantly, following our model of lesson planning, the teacher would have prepared task variations that reduce an aspect of the demand of the original task, rather than changing the task altogether or gathering such students together for a teaching session. Examples of task variations that form a bridge to understanding the original task might be to have available some sheets of the required dimensions–even some with the corner squares already marked–and to invite students experiencing difficulties to cut out the squares and make boxes. Those who experience difficulty even at this level could be given a card that is already marked in a square grid. The intention is that once such students have completed the variation on the task, they will be more likely to be able to proceed with original task as posed, and will be able to both follow and make contributions to the subsequent classroom discussions.

Of course some students will finish the original task before others. We recommend that teachers seek to extend those students' experience with this task, rather than posing something completely different. For example, students who complete a number of solutions could be asked to try to find all possible solutions, to find different ways to represent their answers, to present their findings graphically, and so on. Simple prompts such as "Is there a pattern?" or "What are the largest and smallest volumes possible, and how do you know?" will keep quicker students on task and stimulate some higher-level mathematical thinking.

6 Discussion and conclusion

The above is an example of a classroom task that seeks to engage students in productive exploration of a mathematical situation, by using an open ended problem with a clear mathematical focus. Examples were given of ways that teachers can support the learning of students who experience difficulty as well as of those who complete the task. Specific pedagogical prompts can be used by teachers to support participation of a range of students who may otherwise be marginalised.

In a series of case studies over the past three years, we have found that it is possible to plan and teach lessons structured similarly, that such lessons do engage students in mathematical explorations, and that it is possible for teachers to pose appropriate variations to the tasks. Students generally responded positively to such variations, and when teachers were explicit about the pedagogical intent this improved the students' levels of engagement and the quality of their responses.

We believe that such lessons are possible for the teaching and learning of any mathematics topic at any level. We welcome any feedback from readers on whether they see this as relevant for the level at which they are teaching.

References

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