Book reviews

Proofs from THE BOOK

Martin Aigner and Günter M. Ziegler
3rd Edition
Springer-Verlag Heidelberg 2004
ISBN 3-540-404460-0

Most Australian mathematicians have been privileged to be present at at least one seminar given by the late Paul Erdős. He was a regular, almost frequent visitor to Australia and always gave talks of a very similar style. He would choose a topic such as additive number theory, extremal graphs or Ramsay theory and talk for an hour or so on key results in the area, recent progress (often joint work of Erdős) and unsolved problems in the area. He very rarely talked about details of the proofs. Whatever the topic, he always mentioned money prizes that he personally had offered for solutions to problems. Sometimes he indicated how difficult he thought problems were in other ways. As an example – it is well known that the University of Illinois put "$2^{11213} - 1$ is prime" on its post franking machine in the mid–sixties. Paul suggested that it would be changed to "$2^p - 1$ is prime infinitely often" in the year 5001. Another example he quoted was that if powerful aliens were to arrive and threaten to destroy the planet in 5 years time unless we could supply the value of the Ramsay number $R(5,5)$ then the best strategy would be for us to divert all mathematicians, computer scientists and computers to attempt to calculate the value. On the other hand if they asked for the value of $R(6,6)$ then the best strategy would be to divert all the world's resources into weapons research!

Paul also often talked about proofs that came from "the book" or "the book proof". According to Erdős the Book is "maintained" by God, also known to him as "the great fascist", and contains the perfect proofs for all mathematical theorems. When Erdős said a proof was "from the book" what he meant was that in his opinion the proof was beautiful, striking and probably the most insightful that could be found. He certainly understood that his views were culturally biased and I recall him telling the story that a Sirian mathematician (from Sirius 5) had just received a Sirian Fields Medal for a major contribution to the Riemann Hypothesis – to be precise he had received his award for realising that something so obvious as the location of the zeros of the zeta function actually needed to be proved! For further information on Paul Erdős and his unique life style may I recommend "The man who loved only numbers" by P. Hoffman (London, 1998). Perhaps I should also mention Csiziczery's documentary film on Erdős, now available from Springer on both video and DVD.

In the early 1990s Aigner and Ziegler suggested to Erdős that with his help they should produce a first approximation to a small portion of THE BOOK. It is unusual for a reviewer to have the opportunity to review the first three editions of a book – the first edition was published in 1998, the second in 2001 and the third in 2004. All three editions contain about thirty chapters grouped into five parts – Number Theory, Geometry, Analysis, Combinatorics and Graph Theory. The choice of topics has clearly been influenced by Paul.

Not only do they reflect his view of mathematics but a number of the proofs are either due to him or are improvements of his proofs. The authors also decided to only present Book proofs that could be understood by those who had had exposure to basic analysis, linear algebra, number theory and discrete mathematics. (This is not to suggest that THE BOOK does not contain proofs which use the Cohomology of Vector Bundles of ...)

I was fortunate enough to obtain a copy of the first edition while travelling in Europe in 1999 and I spent many pleasant hours reading it carefully from cover to cover. The style is inviting and it is very hard to stop part way through a chapter. Indeed I have recommended the book to talented undergraduates and to mathematically literate friends. All report that they are captivated by the material and the new view of mathematics it engenders. By now a number of reviews of the earlier editions have appeared and I must simply agree that the book is a pleasure to hold and to look at, it has striking photographs, instructive pictures and beautiful drawings. The style is clear and entertaining and the proofs are brilliant and memorable.

One of the unfortunate decisions that the authors had to make was to drop Chapter 12 of the first edition on "The problem of the thirteen spheres". When I first read the chapter I found it rather surprising that it had been included. The proof clearly depended on some rather delicate calculations and did not look like a BOOK PROOF. In the preface to the second edition, the explanation given is that "the proof turned out to need details that we could not complete in a way that would make it brief and elegant". The third edition contains three new chapters taking the total to thirty five. One of these, on three proofs that $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$ is I think my favourite. How many of us have thought "how can I prove this result without using complex variable techniques?" However, I am afraid that Chapter 24 on shuffling cards looks to me more like a very nice article from the American Mathematical Monthly than a chapter from THE BOOK. Clearly every serious reader of the book will have different views on the choices made by the authors but needless to say that when the fourth edition appears in 2007 I will be the first to go to the bookshop to purchase my own copy.

David Hunt
School of Mathematics, University of New South Wales, Sydney NSW 2052
E-mail: d.hunt@unsw.edu.au

◇ ◇ ◇ ◇ ◇ ◇

Mathematics of Genome Analysis

Jerome K. Percus
Cambridge University Press Cambridge 2002
ISBN 0-521-585171

The author's stated aim in this book is to set forth the mathematical framework in which the burgeoning activity in the "hyperactive field of genomics" takes place. Underpinning his approach is his assertion in the Preface: "that the molecules we have to deal with are fundamentally describable as ordered linear sequences is a great blessing to the quantitatively attuned."[1] They are thus amenable to study by methods of statistical physics and information science.

Chapter 1 introduces nuclear DNA, simply and carefully describing the base pairs, and the representation of double-chain DNA as a linear sequence of the symbols A, T, G and C, with a period often as long as 3×10^9. Genes are characterised as 'words' in these symbols that translate into proteins, and occupy approximately 3% of the sequence. Other components of the DNA

[1] Preface, page ix

sequence of known function are mentioned, with the remainder of the sequence called 'junk DNA' (of unknown function). In reality DNA has an important 3-dimensional spatial or folding structure, and this spatial structure is crucial for the resulting protein. However the linear DNA sequence determines this 3-dimensional structure (although it is only partially clear how), and so the 1-dimensional structure of DNA is the focus of the book. In particular proteins are identified simply as certain subsequences of a DNA sequence.

The initial information sought, therefore, is knowledge of a full linear DNA sequence of period $\sim 3 \times 10^9$. Various fragments of it can be obtained using enzyme cuts and other methods. These fragments, which can be replicated arbitrarily and studied intensively, are called *clones*. The hope is that the full sequence, or *genome*, can be understood by studying a number of clones that together cover it. Elementary probability theory appropriate for studying these clones is introduced and used, for example, to estimate the average distance between special words in the full DNA sequence.

Chapter 2 considers the problem of recomposing a full DNA sequence from a given *fingerprint* consisting of a set of subsequences (clones) that cover the sequence with substantial but unknown overlap. The idea is to order these pieces and identify their overlaps, building up 'islands' of connected overlapping clones. Probabilistic methods exploiting both enumeration and analysis are used. Some technical language about the biological setting at this point made we wish I had been present at the author's lectures in order more easily to gain the necessary understanding. However one can simply 'read past' these remarks in a first reading.

In Chapter 3 it is assumed that long stretches of the DNA sequence are known, and the task is to analyse sequence statistics, such as the distribution of singletons, pairs and various other subsequences. Chapter 4 deals with the situation in which certain words and 'phrases' in a given DNA sequence recur in another DNA sequence. The problem is to estimate the probability that such an event is random and not 'an indicator of a functional or an evolutionary relationship between the chains'. Various techniques are employed to measure the degree of similarity and assess its significance. The final chapter considers briefly the spatial structure and dynamics of DNA. It contains a discussion of various models that have been proposed to describe the large scale behaviour of DNA, especially the way in which DNA transmits its information.

The type of mathematics employed is varied, and is introduced as required. Elementary probability theory is used throughout. Generating functions, and Fourier and Laplace transforms are used to estimate coverage of a sequence by clones. Experimental designs are employed to locate proteins in a long sequence fragment. The theory of random walks is used to build up islands of overlapping clones. Stochastic models such as hidden Markov chains are used to analyse sequence statistics. Spectral methods locate the placement and separation of identifiable subsequences. The theory of entropy gives a measure of the information content of a sequence. Bayesian analysis and neural networks are also used.

There is a substantial bibliography and a useful index. However I found it difficult to keep track of the notation, of which there is a great deal. The book would have benefited from a glossary of notation, both mathematical and biological, and perhaps the author might consider making such a resource available via the web. The weighting given to discussion of the biological background will suit some readers more than others - it will be found by some to be too much, others too little.

The book is a good choice to use for a reading seminar involving a mix of mathematicians and those with some biological training. Each chapter ends with substantial assignments that could be used in such a reading group or in a class-room setting. Professional mathematicians and students wishing to understand what mathematics is being used in Genome Analysis, and how it is used, will find this short book a useful and challenging introduction.

Cheryl E. Praeger
School of Mathematics and Statistics, University of Western Australia
E-mail: praeger@maths.uwa.edu.au

◇ ◇ ◇ ◇ ◇ ◇

Introduction to Banach Algebras, Operators and Harmonic Analysis

H.G. Dales, P. Aiena, J. Eschmeier, K. Laursen and G. Willis
LMS Student Texts **57**
London Mathematical Society 2003
ISBN 0-521-82893-7

This text contains expanded versions of five series of lectures from two instructional conferences, the first in Mussomeli, Sicily in 1999, the second at Sadar Patel University, Gujarat in 2002. The various series are:

- I (7 lectures) (Dales) Banach algebras
- II (5 lectures) (Willis) Harmonic analysis and amenability
- III (8 lectures) (Eschmeier) Invariant subspaces
- IV (5 lectures + appendix) (Laursen) Local spectral theory
- V (3 lectures) (Aiena) Single-valued extension property and Fredholm theory

In broad terms each series gives a brief introduction to the indicated area by a relevant expert. (Several of the writers have (co)authored major monographs on the indicated area; for the reader's information these are listed below.) The basic material is in each case leavened by more recent results, and each lecture contains exercises and additional notes. Although the individual series can be read independently, there are underlying themes which recur throughout giving consistency to the overall volume. The level is suitable for a beginning graduate student, yet the approaches give good indications of the flavour of current work in the various areas. The volume is nicely produced, with few misprints, though the incorrect page header for Chapter 6 and a missing reference in Part I are unfortunate. Overall, the volume is highly recommended as an accessible introduction to several important areas of modern functional analysis.

To give rather more detail, recall that a Banach algebra is an algebra (almost always taken over $\mathbb{C}$), with a norm topology under which the algebra is complete as a vector space, and with respect to which multiplication is continuous (separately or jointly are the same by the uniform boundedness principle). The study of such objects has developed into a major theory in its own right, as well becoming a basic tool in other areas of functional analysis.

Part I is an introduction to the general theory of Banach algebras and important examples, and in particular gives the necessary background for the other parts. It includes some material on 'automatic continuity' where the interplay of algebraic and topological properties is to the fore.

Part II is concerned with the particular example of the convolution algebra $L^1(G)$ for G a locally compact group. As well as discussing standard constructions, it includes material (much of it due to the author) on the structure of ideals of finite codimension and their link to automatic continuity questions.

The fact that a linear transformation on a finite dimensional vector space has an eigenvector is an important ingredient of the structure theory for matrices. The corresponding infinite dimensional question, as to whether a bounded linear operator on a Banach space has a non-trivial closed invariant subspace is very much more difficult, indeed it fails in general, but the Hilbert space case remains an open question after 80 years of study. Part III discusses these questions, and in particular one approach that has yielded positive results for suitable operators.

Part IV discusses local spectral theory, which in very broad terms considers the situation when decompositions of the spectrum of an operator give rise to invariant decompositions of the Banach space on which the restricted operators have spectrum contained in the corresponding piece of the original spectrum. The "decomposable" operators with this property form a large class with very important properties. For example, they include compact operators on a Banach space, and normal operators on a Hilbert space. Surprisingly, regularity of a commutative semisimple Banach algebra is equivalent to decomposability of the multiplication operators!

Part V continues with further discussion of the single valued extension property (already introduced in Part IV) and semi-Fredholm operators.

References

[1] H.G. Dales, *Banach algebras and automatic continuity* London Mathematical Society Monographs **24** (Oxford Science Publications, The Clarendon Press, Oxford University Press New York 2000).

[2] K.B. Laursen and M.M. Neumann, *An introduction to local spectral theory*, London Mathematical Society Monographs **20** (The Clarendon Press, Oxford University Press New York 2000).

[3] P. Aiena, *Fredholm and local spectral theory, with applications to multipliers* (Kluwer Academic Publishers Dordrecht 2004).

R.J. Loy
Mathematical Sciences Institute, Australian National University, Canberra ACT 0200
E-mail: Rick.Loy@maths.anu.edu.au

◇ ◇ ◇ ◇ ◇ ◇

Essentials of Mathematics: introduction to theory, proof and professional culture

Margie Hale
MAA Washington 2003
ISBN 0-88385-729-4

This beautifully written book is intended to be used by first or second year undergraduate university students to bridge the gap from school mathematics to 'what comes next'. The author has designed the book to be used as a text that introduces students to 'language and methods of the axiomatic system and the art of proof'. She expects students to work through the book guided by a 'knowledgeable, enthusiastic and caring instructor'.

The first section of Chapter 0 lists several outcomes that the author hopes students undertaking such a course will gain. In addition to several outcomes relating to the content, these outcomes include 'practice in constructing proofs and evaluating the proofs of others', 'an introduction to the professional culture inhabited by mathematicians', and lastly 'an eagerness to do more mathematics'.

Students who complete the first four chapters will be prepared for theoretical courses in linear algebra, abstract algebra, real and complex anaysis, and topology. They will be familiar with many of the techniques of these courses. In addition Chapter 5 contains the beginings of a course on real analysis. The book grew out of the author's 'Moore method' philosophy of teaching, but the author has tried to make it adaptable to any style of teaching, and gives several examples of how it might be used.

The 'course material' consists of definitions, theorems, and exercises, with no answers or hints, more details of which will be given below. Each chapter begins with a background discussion and some 'warm-up exercises' intended to encourage students to ask questions and to engage in the concepts they will meet more formally later in the chapter. This is followed by an 'essentials' section containing the concepts and results. Then follows a section of 'further exercises' that may be used by students for review, to summarise the material, or to clarify some of the concepts introduced in the chapter. The treatment is careful and succinct. The final sections of each chapter present material about 'mathematical culture'. In some chapters this is a discussion of famous mathematical theorems or conjectures, while in one it is a discussion of ethics in mathematical writing. The underlying aim is to give undergraduate students knowledge of things that every educated mathematics graduate should know, but 'are not found in formal coursework, except by chance'.

In Chapter 0 the author tries to describe what mathematicians do, and what they find satisfying. She recounts a joke heard at an 'annual math meeting':

> Q: How do you tell an introverted mathematician from an extroverted mathematician?
>
> A: An extroverted mathematician looks at *your* shoes while he's talking to you.

wryly commenting on the joke's assumption that mathematicians are male, and then proceeds to sketch perceptively the diversity of mathematicians and various of their traits. This chapter also contains a very nice discussion of the connections and distinctions between pure mathematics and applied mathematics.

Chapter 1 presents the essentials of logic. Its final sections describe the axiomatic method and results of Gödel. The introduction to Chapter 2 tackles the question 'What is proof?'. The author distinguishes between the processes of discovery and public presentation of a proof; she analyses the structure of a proof and discusses strategies to help a student both construct and read a proof. The essentials of this chapter are from set theory, and the final section discusses several famous paradoxes, the Zermelo-Fraenkel axioms and the Axiom of Choice. Chapter 3 starts by discussing the use of symbols. The essentials of this chapter include Peano's axioms for the natural numbers, and various of their properties including the axioms for order. The chapter concludes with a brief exposition of cardinal arithmetic and ordinal numbers, and includes Cantor's diagonalization argument. Chapter 4 begins with a discussion of the philosophy of mathematics, addressing in particular the question of whether mathematics is created or discovered. The essential content is the construction and properties of the positive rational numbers. The chapter ends with a discussion of ethics, especially regarding the writing, presenting, and reviewing of mathematics research. In Chapter 5, Dedekind cuts are introduced to construct the positive reals. A formal construction of the set of all real numbers is given and its properties are explored, including order, completeness, and density of the rationals. The final section looks forward to limits and calculus.

The last two chapters diverge from the general pattern. Chapter 6 begins with thumbnail sketches of the contributions to mathematics of 27 famous mathematicians, from Newton to Erdös and Knuth. The paragraph-long summaries are delightful and many contain references for further reading. The essentials of this chapter are properties of the complex numbers. The chapter ends with discussion of the Fundamental Theorem of Algebra, the complex plane, and various applications of complex analysis. The last chapter addresses the challenging question: 'What is mathematical research?' The importance of asking good questions is stressed, and examples of how to do this are given through considering problems arising from the game of Nim and related games. The book ends with brief descriptions of several famous theorems and unsolved problems, and a (predominantly US-based) overview of professional mathematical organisations and resources.

This is a wonderful book. Although it is most suitable for use within the US course structure, it is a thought-provoking resource for anyone involved in curriculum development and teaching of mathematics at a university entrance level. By way of comparison, every engineering undergraduate program, at least in my country, includes an introductory course on professional engineering. A course embracing the cultural aspects of this book, along with the broad mathematical content, would fulfil the same role for an undergraduate program in the mathematical sciences. I commend it to my mathematical colleagues and to students of mathematics, and thank the author for writing it. It should be essential reading for *every* enthusiastic mathematics undergraduate.

Cheryl E. Praeger
School of Mathematics and Statistics, University of Western Australia
E-mail: praeger@maths.uwa.edu.au

◇ ◇ ◇ ◇ ◇ ◇

Combinatorics of Permutations

Miklós Bóna
Chapman and Hall/CRC 2004
ISBN 1-58488-434-7

This book is a timely and welcome introduction to the field of "permutation patterns", as it is sometimes called, in combinatorics. Consider a permutation as an arrangement of a set of distinct numbers, say $1, 2, \ldots, n$. A permutation is said to *contain* a shorter permutation, or "pattern", if some choice of entries in the first permutation has the same relative ordering as the entries of the second, and *avoids* a pattern otherwise. For example, 52314 contains the pattern 321, by taking the entries $5, 3, 1$, but avoids the pattern 132.

Bóna examines a wide range of problems arising from counting permutations under various restrictions. He devotes a chapter (Chapter 4) to the motivation and resolution of a major open conjecture in this field: the Stanley-Wilf conjecture. The number of permutations of length n is of course $n!$, but the Stanley-Wilf conjecture asserted that as soon as you restrict to permutations that avoid at least one subpattern, the number drops down to c^n for some constant c. The beautiful and surprisingly "elementary" proof of Marcos and Tardos is presented at the end of the chapter; Zeilberger refers to this proof as one "from the book" [1].

The next chapter concerns another important conjecture for the field, credited to Gessel, Noonan and Zeilberger, about the nature of generating functions for sets of pattern-avoiding permutations. The concepts of P-recursive sequences and algebraic and D-finite generating functions are explained. The connections between Standard Young Tableaux and permutations are also discussed in detail, including an exposition of the Robinson-Schensted-Knuth correspondence.

The first three chapters bring together a solid background in the study of permutations from various viewpoints, and the later chapters are concerned with pattern avoidance or containment (for instance, how many copies of a certain pattern can a permutation contain, and which permutations realise the maximum packings?). A natural place that pattern avoidance arises is in stack-sorting, and a reasonable treatment of the major results to date is included.

The book is readily accessible to honours and graduate students, and also serves as a useful resource for anyone working in combinatorics. Bóna's style is relaxed and engaging, his proofs are often very visual and combinatorial, and he spends time looking carefully at particular examples rather than giving sweeping overviews. Each chapter ends with exercises and harder "Problems Plus" which have solutions.

Its contribution to this relatively new field is that it brings together and makes very clear the motivations, open problems and different approaches that have been used by researchers previously.

References

[1] http://www.math.rutgers.edu/$\sim$zeilberg/Opinion58.html

Murray Elder
School of Mathematics and Statistics, University of St Andrews, Scotland
E-mail: murray@mcs.st-and.ac.uk