Extended Citation - Dr Kevin Coulembier

Dr Kevin Coulembier is an international leader in Lie theory and representation theory, who has made outstanding contributions to the field by solving several important and prestigious problems, which remained open for many decades.

His research has been recognised with a medal from the Australian Academy of Science, an ARC Future Fellowship, a DECRA and two Discovery Projects. He has an impressible publication record with many publications in top international journals and receives several invitations to speak at prominent international conferences every year. He is also active as journal editor and conference organiser.

Below is a description of some of Kevin's work, which was carried out in Australia, in the research areas of tensor categories, BGG category $\mathcal{O}$ and primitive ideals.

Tensor categories I: Structure theory

The study of tensor categories was initiated by Grothendieck in order to establish bridges between various areas of mathematics. Tensor categories are specific abelian monoidal categories defined over a base field k, purposely modelled on representation categories of affine group schemes, or more generally groupoids acting on a scheme.

A problem of critical importance in the subject is finding intrinsic criteria for a tensor category to be tannakian, that is, to be equivalent to such a representation category. When the base field k has characteristic zero, this was solved by Fields medal winner Deligne in 1990. The corresponding problem in prime characteristic $p > 0$ proved to be very challenging, and remained unsolved ever since. In paper [13][1] published in *Duke Mathematical Journal*, Kevin obtained the solution to this problem as part of a larger original treatment of tensor categories in positive characteristic.

The original aim of the theory of tannakian categories was to apply techniques of representation theory to other areas involving the study of, e.g., categories of motives of projective varieties or of locally constant sheaves, which are known be tannakian. Kevin's work therefore now opens the possibility of applying such techniques in the case of positive characteristics.

Over fields of characteristic 0, Deligne proved that any reasonable tensor category which is not tannakian is 'super' tannakian, which means it is a representation category of a super group. This has been known to be false over fields k of positive characteristic $p > 0$ since the '90s. In paper [1], published in *Annals of Mathematics* and building further on [13], Kevin finally proved a suitable analogue of this celebrated result in collaboration with Etingof and Ostrik. Under the technical assumption of 'Frobenius exactness' which was satisfied in all tensor categories known until 5 years ago, they proved that Deligne's result remain valid provided one replaces the category of super vector spaces with a larger tensor category Ver_p.

Despite being so recent, this breakthrough has already led to a variety of fascinating applications in modular representation theory, for instance on growth rates of tensor powers and classification problems. This work will also feature in the June 2023 session of the prestigious *Séminaire N. Bourbaki*, futher highlighting the impact these results have made.

Tensor categories II: Abelian envelopes

Kevin leads the development of the theory of abelian envelopes, which are tensor categories associated to non-abelian monoidal categories via a universal property. This is the main method for

[1] references [1], [2], . . . refer to the corresponding papers in Kevin's publication list

constructing new tensor categories. In the paper [7] in *Compositio Mathematica*, Kevin established the first criteria for the existence of abelian envelopes, and obtained a construction for them. This recovered all existing abelian envelopes obtained by other authors. More important, Kevin's construction yielded new examples, which provided the first examples of tensor categories which are *not* Frobenius exact. These were at first only discovered in characteristic $p = 2$ by Benson and Etingof. These newly discovered tensor categories, labelled Ver_{p^n}, $n \geq 1$, lie at the heart of the current research in tensor categories. Indeed, understanding these categories from [7] seems the gateway to a full understanding of the structure theory of tensor categories, a subject which has now been active for over 30 years.

As demonstrated in recent developments [3, 4], Kevin's work opens new applications of the theory of abelian envelopes. Two concrete examples are given by the important open question, posed in 1990 by Deligne, of whether the extension of scalars of a tensor category is always a tensor category, and very explicit conjectures about the existence of abelian envelopes which Kevin and collaborators already verified for tannakian categories in paper [3], published in *Journal für die Reine und Angewandte Mathematik.*

In another direction, in paper [2], published in *Selecta Mathematica*, Kevin introduced a generalised notion of abelian envelopes and proved that these (contrary to actual abelian envelopes) always exist. In particular, this vastly enlarges the potential for constructing new tensor categories.

Bernstein-Gelfand-Gelfand Category $\mathcal{O}$

The BGG category $\mathcal{O}$ associated to complex reductive Lie algebras is one of the most influential and foundational parts of modern representation theory, and has been intensively studied ever since its introduction in the '70s. Several top mathematicians, such as Jantzen, Stroppel and Soergel, had observed that there exist many equivalences between different indecomposable categories appearing therein. In [15], Kevin completed the classification of all such equivalences, concluding this line of research, which had been ongoing for over 40 years. This not only led to a complete explicit classification of the blocks up to equivalence, but to an original new characterisation of any block by a single partially ordered set. Particularly interesting to representation theorists was Kevin's astonishing discovery, applied as a crucial part of the proof, that highest weight structures in categories with duality are unique. This is applicable to many areas of representation theory, for instance modular representation theory and Kac-Moody algebras.

Primitive ideals of superalgebras

The description of the primitive spectrum of a reductive Lie algebra (the classification of the annihilator ideals in the universal enveloping algebra of the simple modules, along with the inclusion order) was finalised in the '80s, by work of Vogan, Joseph, Duflo, Borho and Jantzen, and has interesting links to Kazhdan-Lusztig combinatorics and various aspects of representation theory. For Lie superalgebras, Musson proved already in 1992 that the question reduces, similarly as for Lie algebras, to the question of which inclusions appear between annihilator ideals for simple representations in category $\mathcal{O}$. Since then, several authors, such as Letzter, Mazorchuk and Musson obtained partial answers, but the full problem remained open for over 20 years.

In paper [31] in *Communications in Mathematical Physics*, Kevin concluded a long term research project, partly in collaboration with Mazorchuk and Musson, which completely described the primitive spectrum in terms of super Kazhdan-Lusztig combinatorics.