

## **The Szekeres Medal Citation for Igor Shparlinski**

The George Szekeres Medal is the Australian Mathematical Society's most prestigious medal, recognising research achievement and an outstanding record of promoting and supporting the discipline. Igor Shparlinski's long contributions over a long time make him a deserving winner of the Szekeres medal.

Igor has made notable contributions in many areas of mathematics with specialisms in number theory and its applications

Since his move to Australia in 1992, Professor Shparlinski has supervised 15 PhD, 10 Masters and 25 Honours students to completion, as well as mentored 15 postdocs in their progression towards career independence. Many of his former students and postdocs have subsequently received prestigious fellowships and appointments. When Igor arrived at UNSW in 2013 research in Number Theory there was in decline, after having been first established in the department by George Szekeres. He rebuilt a strong Number Theory group at UNSW and number theory is now one of the strongest research discipline areas in Pure Mathematics at the University. This growth in Number Theory at UNSW has achieved broader impact across the Australian mathematical community via a strong visitor program.

The influence of Shparlinski on mathematics is reflected in his incredible citation rate. For example, Scopus finds nearly 5000 citations to his over 600 papers and calculates his h-index as 29.

### **Research achievements.**

Professor Shparlinski's most significant results are based on a broad range of ideas and approaches – often taken from very different fields and combined in novel ways. His arsenal of methods includes the following deep and complex technical tools:

- Bounds on short exponential and character sums; bounds of incomplete and bilinear Kloosterman sums; and Vinogradov's Mean Value Theorem.
- Sum-product theorems, incidence and energy bounds and other methods and results of additive combinatorics.
- Diophantine geometry such as the theory of heights, effective forms of Hilbert's Nullstellensatz, and Hilbert's Irreducibility Theorem.
- Results on numbers with prescribed arithmetic structures (such as primes and smooth numbers) obtained via sieves and other methods of analytic number theory.

Professor Shparlinski's bound on high degree Gauss sums was the first example of a bound on such sums below the 'square-root' threshold. As an application, he used it to settle Stechkin's conjecture about these sums. These results initiated a new line of research pursued by a number of prominent mathematicians from around the world. The initial motivation for this work came from establishing *the Holden-Moree conjecture* on the discrete logarithm in finite fields (which has a strong cryptographic aspect). However, the main value of these results is in their further applications to a broad range of other well-known problems on pseudo-powers, Fermat quotients,  $g$ -ary digits of rational fractions, etc. There are now several research groups who continue working in this direction.

About a decade ago, Professor Shparlinski initiated a new direction, named “Unlikely intersections in finite fields”. This designation reflects the actual content: establishing the mutual avoidance between different types of arithmetic and algebraic structures. This work used a diversity of methods from additive combinatorics, analytic number theory, Diophantine geometry and arithmetic dynamics. The ideas and results of his work with provided the origins for applications in addressing a number of problems in cryptography and algorithms.

Shparlinski has obtained new results on cancellations in families of important exponential and character sums, such as Kloosterman sums. In particular, he has improved several previous results in the area on fourth moments of  $L$ -functions. Most recently, Professor Shparlinski started working on bounds and applications of *Weyl sums*. He has used the ideas to derive some results towards introducing a new direction in the theory of these sums, interpolating between the optimal bound on the *average values* of Weyl sums - known as Vinogradov’s Mean Value Theorem -and *individual bounds* where the results are far from what is believed to be optimal.

### **Promoting and supporting the discipline.**

Igor has a notable record in promoting and supporting the discipline. As mentioned above, he has successfully supervised 15 PhD students and ten masters students. Igor has also mentored fifteen postdoctoral fellows who have gone on to obtain further postdoctoral positions or continuing positions.

Professor Shparlinski is an editor of the book series *Frontiers in Mathematics* (Birkhauser) and has served (and continues to serve) on several editorial boards. Most significantly, he is on the Editorial Committee of *Mathematics of Computation*; the body of four mathematicians deciding the long-term policy and strategy of this journal. He is also a regular invitee and participant of workshops in such highly prestigious centres as Oberwolfach, Dagstuhl, CIRM-Luminy, BIRS-Banff, and the Fields institute.

### **Summary**

Professor Igor Shparlinski is one of Australia’s leading mathematicians, specialising in number theory and its applications (including to cryptography). Seminal amongst his achievements are: new bounds of exponential and character sums; developing the metric theory of Weyl sums; and developing the theory of “unlikely intersections” in finite fields using the methods of Diophantine geometry and additive combinatorics. Moreover, he has made influential discipline leadership contributions through the establishing of a world class Number Theory group in Australia, the membership of the Editorial Board of several prestigious international journals and the oversight and organisation of several national/international conferences. He has built capacity via the supervision, mentoring and the attracting of many outstanding Number Theory researchers to visit Australia.