Obituary: Vaughan Jones

From: Arnaud Brothier, Pinhas Grossman, Scott Morrison, and James Tener

Vaughan Jones, one of the world's outstanding mathematicians, passed away suddenly on 6 September 2020, at the age of 67. Born and raised in New Zealand, Jones was the first mathematician from the southern hemisphere to be awarded a Fields Medal. Known for an unconventional and informal style, he delivered his plenary lecture at the International Congress of Mathematicians in Kyoto in 1990 wearing an All Blacks rugby jersey.

Jones' primary field of research was the theory of operator algebras, a mathematical discipline initiated by John von Neumann in the early 20th century and inspired by the discovery of quantum mechanics. But he carved out his own distinctive path, cutting across a vast and diverse mathematical landscape. His pioneering work and extraordinary vision have had enormous influence on a wide range of different areas of mathematics and theoretical physics, from low-dimensional topology to quantum field theory. Longtime friend and colleague Roberto Longo likened Jones' unique mathematical style to that of a painter, producing incredibly beautiful and deep works of art with the simplicity of a great master.

Vaughan Frederick Randal Jones was born on 31 December 1952 in Gisborne, NZ. He was educated at St Peter's School in Cambridge, NZ and Auckland Grammar School. After undergraduate studies at the University of Auckland, he went to Switzerland where he completed a PhD at the University of Geneva under the supervision of André Haefliger and Alain Connes. He moved to the United States in 1980, where he held positions at the University of California, Los Angeles and the University of Pennsylvania, before arriving at the University of California, Berkeley. He was a Professor of Mathematics at Berkeley for 26 years. In 2011, he retired from Berkeley as Emeritus Professor and moved to Vanderbilt University as Stevenson Distinguished Professor of Mathematics, where he remained until his death.

In the 1980s, Jones initiated the theory of subfactors. A factor is a maximally noncommutative von Neumann algebra, i.e. one with trivial center. A subfactor is an embedding of one von Neumann algebra inside another. Subfactor theory is concerned with symmetries arising from such embeddings. In his PhD thesis, Jones had classified ways in which a finite group can act on a factor; such group actions give rise to subfactors of fixed points, whose symmetries reflect the acting groups. Motivated by an analogy with subgroups of groups, he then defined an index for arbitrary subfactors, and proved the surprising result that this index is "quantized" - only a discrete spectrum of values between 1 and 4 can be realized as the index of a subfactor (although all values above 4 are realized).

Jones is perhaps most widely known to general mathematical audiences for a knot invariant called the Jones polynomial. The classification of knotted strings in three-dimensional space is a fundamental problem in topology. The Jones polynomial is a powerful invariant that is able to distinguish between some knots that were not distinguished by earlier invariants such as the Alexander polynomial, which had been discovered in the 1920s. The Jones polynomial led to the resolution of several of the Tait conjectures from the 19th century, and has found modern applications as far afield as understanding the structure of knotted DNA strands.

What was particularly astonishing about the discovery of the Jones polynomial is that it emerged from the seemingly unrelated field of operator algebras. While proving the index theorem, Jones had

found that the symmetries associated with subfactors give rise to certain algebraic relations first observed in the context of statistical mechanics. These relations appear similar to those satisfied by canonical generators of the braid group, which led Jones to construct new representations of the braid group. He then showed that a trace on these representations coming from the von Neumann algebra setting could be used to define a knot invariant.

The discovery of the Jones polynomial led to a surge of activity in a number of diverse research areas - including quantum groups, statistical mechanics, topological quantum field theory, and low-dimensional topology - and launched an entirely new field called quantum topology.

Aside from its applications to knots and mathematical physics, Jones' theory revealed intricate and beautiful internal symmetries of subfactor embeddings that have come to be known as "quantum symmetries" (following the analogy of subfactors with groups). For subfactors with small index - up to 4 - these symmetries are always related to groups or quantum groups. But in the 1990s new "exotic" quantum symmetries associated with subfactors with certain indices slightly larger than 4 were discovered. These mysterious quantum symmetries had not appeared in any other mathematical context, and therefore represent genuinely novel mathematical phenomena.

One of Jones' major projects in the late 1990s and early 2000s was his theory of planar algebras. Originally conceived as a topological axiomatization of the standard invariant of a subfactor, the notion of a planar algebra has found broader application as a concrete formalism for analysing the structure of pivotal categories, which play an important role in representation theory. As Terence Tao has written, "Traditional algebra notation is one-dimensional in nature, with algebraic expressions being described by strings of mathematical symbols... Planar algebras, by contrast, fully exploit the topological nature of the plane... I found the mere possibility of higher dimensional notation being the natural choice for a given mathematical problem to be conceptually liberating."

When Jones first arrived in Geneva in 1974, he began studying physics, and only subsequently switched to mathematics for his PhD. But physical motivation remained a guiding constant throughout his career, and the ideas and insights flowed in both directions. His subfactor theory plays a foundational role in the conformal net approach to conformal field theory. More recently, subfactor theory has seen increasing connections with both conformal field theory and topological quantum computing, an approach to quantum computation pursued at Microsoft's Station Q by a group led by Fields Medallist topologist Michael Freedman.

In his later years, Jones frequently cited as inspiration the mathematical problem of determining whether every subfactor arises from conformal field theory in some suitable sense. This problem is still open, and is being studied from different points of view by a number of his colleagues and students. While he did not solve the problem, in typical fashion Jones' inquiries led to a wealth of interesting and original mathematics, including new unitary representations of the famous Thompson's group - which in turn led to novel connections between Thompson's group and knot theory.

His remarkable career earned Jones numerous accolades and awards, in addition to the Fields Medal. He was inducted as a Fellow of the Royal Society in 1990 and as a member of the National Academy of Sciences in 1999. In an unusual honor for a mathematician, he was made Honorary Vice President for Life of the International Guild of Knot Tyers in 1992. He was designated a Knight Companion of the New Zealand Order of Merit in 2009. In 2010 the Royal Society Te Apārangi established the Jones Medal in his honor.

He also held numerous positions of leadership in the discipline, including Vice President of the American Mathematical Society and Vice President of the International Mathematical Union. He had a lifelong commitment to supporting mathematics in New Zealand, and was principal founder and leader of the New Zealand Mathematics Research Institute (NZMRI).

Jones also had close connections to Australia. Elected as a Corresponding Fellow of the Australian

Academy of Science in 1992, he visited Australia a number of times, including most recently in 2019 for the workshop Subfactors in Sydney at UNSW. Australian mathematicians were regularly featured in the annual NZMRI summer schools for graduate students that Jones organized. Four of his former PhD students now work in Australia, on a range of topics including higher category theory, conformal field theory, subfactor theory, and representations of Thompson's group - reflecting Jones' broad research interests.

Vaughan was a colleague, mentor, and friend to countless mathematicians around the world, including more than 30 PhD students over the course of his career. A humble, kind, and down-to-earth person with a great sense of humor, Vaughan was never one to stand on ceremony. He was remarkably approachable for such a formidable figure, and was always particularly gracious in his support of students and junior researchers.

As his former students, we benefited from and were inspired by his generosity with his time and ideas, as well as his encouragement in developing our independent research directions. A highlight of our PhD years was Vaughan's subfactor seminar at Berkeley, later at Vanderbilt, where the freewheeling exchange of ideas made for an exhilarating experience. Vaughan set the tone, never shy about challenging a speaker or asking for additional explanation, with the resulting interaction invariably elevating the discussion.

Vaughan's generosity and excitement in sharing mathematical discovery was a hallmark of his research style. As longtime friend and colleague David Evans noted, since the beginning of his career, Vaughan was known in the mathematics community for an "openness in sharing ideas through every stage of development from speculation and conjecture of the way forward to discussing and explaining results" - an unusually inclusive approach in the sometimes competitive world of academic research. His brilliance, charisma, and magnanimity formed a high tide which lifted all who were fortunate to be in his mathematical orbit. And his many mathematical and professional collaborations often turned into close and lasting friendships.

Vaughan was a person of diverse interests who enjoyed life. His many outdoor pursuits included playing rugby in his youth, skiing (which is how he met his wife Wendy in Geneva), playing tennis with his family, and golfing in courses around the world in his later years. He was an avid windsurfer and later kitesurfer, which often led him to coordinate his travel schedule with the sea and the wind. He had a lifelong involvement in music and singing, and performed for many years in the orchestra at UC Berkeley commencement ceremonies. He was passionate about good coffee, and seemed more proud of his Certificate of Barista Skills than of his Fields Medal.

His students fondly remember and treasure the post seminar beer-and-pizza nights at Raleigh's Pub in Berkeley; weekends at Vaughan's Bodega Bay vacation home ("The Bodega Bay Institute of Mathematics"); summer conferences in Maui and in Vaughan's native New Zealand; and numerous conferences and research programs around the world, the last of which was the 2020 MSRI semester program Quantum Symmetries which Vaughan co-organized.

Vaughan is survived by his wife Martha (Wendy); children Bethany, Ian, and Alice; and grandchildren. He is greatly missed by his family, his many friends around the world, and his students and colleagues.